Learning about the Neighborhood: Supply Elasticity and Housing Cycles*

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Abstract

Motivated by an intriguing observation during the recent U.S. housing cycle that counties with housing supply elasticities in an intermediate range experienced the most dramatic price booms and busts, this paper develops a model to analyze information aggregation and learning in housing markets. In the presence of pervasive informational frictions, housing prices serve as important signals in the households' learning of the economic strength of a neighborhood. Supply elasticity affects not only housing supply but also the informational noise in the price signal. Our model predicts that the housing price and the share of investment home purchases exhibit the greatest variability in areas with intermediate supply elasticities, which is supported by our empirical analysis.

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Conventional wisdom posits that supply elasticity attenuates housing cycles. As a result, one expects housing prices to be more volatile in areas with more inelastic housing supplies. As noted by Glaeser (2013) and other commentators, however, during the recent U.S. housing cycle in the 2000s, some areas such as Las Vegas and Phoenix experienced more dramatic housing price booms and busts, despite their relatively elastic housing supply, compared to areas with more inelastic supply, such as New York and Los Angeles. Interestingly, by systematically examining the cross-section of the booms and busts experienced by different counties during this housing cycle, we find that the monotonically decreasing relationship between the magnitude of housing cycles and supply elasticity is more fragile than commonly perceived. If one simply sorts counties into three groups based on Saiz's (2010) widely-used measure of supply elasticity, each with an equal number of counties, the average housing price increase in the boom period of 2004-2006 and drop in the bust period of 2007-2009 is monotonically decreasing across the inelastic, middle and elastic groups. As the inelastic group holds more than half of the population, however, this coarse grouping may disguise non-monotonicity under present finer parsings. Indeed, when we sort the counties into ten elasticity groups, each with an equal number of counties, or into either three or ten elasticity groups each with an equal population, we uncover a non-monotonic relationship between the magnitudes of the housing price booms and busts experienced by different counties and their supply elasticity. The most dramatic boom and bust cycle occurs in an intermediate range of supply elasticity.

This humped-shape relationship between housing cycle and supply elasticity, which we summarize in Section 1, is intriguing and cannot be explained by the usual supply-side mechanisms. In this paper, we develop a theoretical model to highlight a novel mechanism for supply elasticity to affect housing demand through a learning channel. We emphasize that home buyers observe neither the economic strength of a neighborhood, which ultimately determines the demand for housing in the neighborhood, nor the supply of housing. In the presence of these pervasive informational frictions, local housing markets provide a useful platform for aggregating information. This fundamental aspect of housing markets, however, has received little attention in the academic literature. It is intuitive that traded housing prices reflect the net effect of demand and supply factors. Supply elasticity determines the weight of supply-side factors in determining the housing prices, and therefore by extension determines the informational noise faced by home buyers in using housing prices as signals for the strength of demand.

Our model integrates the standard framework of Grossman and Stiglitz (1980) and Hellwig (1980) for information aggregation in asset markets with a housing market in a local neighborhood. This setting allows us to extend the insights of market microstructure analysis to explore the real consequences of informational frictions in housing markets. In particular, our model allows us to analyze how agents form expectations in housing markets, how these expectations interact with characteristics endemic to a neighborhood, and how these expectations feed into housing prices.

We first present a baseline setting in Section 2 to highlight the basic information aggregation mechanism with each household purchasing homes for their own consumption, and then extend the model in Section 3 to further incorporate purchases of investment homes. The baseline model features a continuum of households in a closed neighborhood, which can be viewed as a county. Each household in the neighborhood has a Cobb-Douglas utility function over its goods consumption and housing consumption, as well as housing consumption of its neighbors. This complementarity in the households' housing consumption motivates each household to learn about the unobservable economic strength of the neighborhood, which determines the common productivity of all households and thus their housing demand.

Despite each household's housing demand being non-linear, the Law of Large Numbers allows us to aggregate their housing demand in closed-form and to derive a unique log-linear equilibrium for the housing market. Each household possesses a private signal regarding the neighborhood common productivity. By aggregating the households' housing demand, the housing price aggregates their private signals. The presence of unobservable supply shocks, however, prevents the housing price from perfectly revealing the neighborhood strength and acts as a source of informational noise in the housing price.

Our model also builds in another important feature that households underestimate supply elasticity. By examining a series of historical episodes of real estate speculation in the U.S., Glaeser (2013) summarizes the tendency of speculators to underestimate the response of housing supply to rising prices as a key for understanding these historical experiences. In our model, underestimation of supply elasticity implies that households underestimate the amount of informational noise in the observed price signal, which in turn causes the households' expectations of the neighborhood strength and housing demand to overreact to the housing price. The amplification of housing price volatility induced by such overreaction depends on the uncertainty faced by households and the informational content of the price, both of which are endogenously linked to the neighborhood's supply elasticity.

It is useful to consider two polar cases. At one end with the supply being infinitely inelastic, the housing price is fully determined by the strength of the neighborhood and thus perfectly reveals it. At the other end, with housing supply being infinitely elastic, the housing price is fully determined by the supply shock and households' uncertainty about the strength of the neighborhood does not interact with the housing price. In between these two polar cases, the households face uncertainty regarding the neighborhood strength and the uncertainty matters for the housing price. Consequently, households' overreaction to the price signal has the most pronounced effect on their housing demand and the housing price in an intermediate range of supply elasticity, causing the price volatility to have a humped-shape relationship with supply elasticity. That is, housing price volatility is largest at an intermediate supply elasticity rather than when supply is infinitely inelastic. This key insight helps explain the aforementioned empirical observation that during the recent U.S. housing cycle, counties with supply elasticities in an intermediate range experienced the most dramatic price booms and busts.

We further extend the baseline model in Section 3 to incorporate immigrants who are attracted to the neighborhood by its economic strength in a later period, and the speculation of the current households in acquiring secondary homes in anticipation of selling/renting them to immigrants. This model extension generates two additional predictions. First, the households' learning effects can induce another non-monotonic relationship between the variability of the share of secondary home purchases among total home purchases and supply elasticity. The intuition is similar to before. As secondary home purchases are more sensitive than primary home purchases to the households' expectations of the neighborhood strength, informational frictions and the households' overreaction to the price signal make households' secondary home purchases most variable at an intermediate range of supply elasticity. This mechanism also leads to a second prediction regarding a positive relationship between the variability of the share of secondary home purchases and housing prices across neighborhoods with different elasticities.

Interestingly, we are able to confirm these new model predictions in the data. First, we find that counties in an intermediate range of supply elasticity indeed had the largest change in the share of non-owner-occupied home (investment home) purchases from the pre-boom period of 2001-2003 to the boom period of 2004-2006, as opposed to counties with either the most elastic or inelastic supplies. Second, counties with greater increases in the share of non-owner-occupied home purchases from 2001-2003 to 2004-2006 also experienced larger price increases in 2004-2006 and larger price decreases in the bust period of 2007-2009. These empirical findings provide evidence from a new dimension to support the important roles played by informational frictions and household learning in driving housing cycles.

The existing literature has emphasized the importance of accounting for home buyers' expectations (and in particular extrapolative expectations) in understanding dramatic housing boom and bust cycles, e.g., Case and Shiller (2003); Glaeser, Gyourko, and Saiz (2008); and Piazzesi and Schneider (2009). Much of the analysis and discussions, however, are made in the absence of a systematic framework that anchors home buyers' expectations on their information aggregation and learning process. In this paper, we fill this gap by developing a model for analyzing information aggregation and learning in housing markets. By doing so, we are able to uncover a novel effect of supply elasticity, beyond its role in driving housing supply, in determining the informational content of the housing price and households' learning from the price signal. This learning effect implies non-monotonic patterns of housing price volatility and share of investment home purchases across neighborhoods with different supply elasticities, which are observed in the data. This learning mechanism also differentiates our model from Gao (2013), which shares a similar motivation as ours to explain the dramatic housing price booms and busts in the 2000s experienced by areas with intermediate supply elasticities, and which emphasizes a joint effect of time-to-build and housing speculators' extrapolative expectations as an explanation.

In our model, households overreact to the housing price signal. Such overreaction is driven by their underestimation of supply elasticity. This overreaction mechanism, which depends on the informational frictions faced by households and the endogenous informational content of the housing price, is different from the commonly discussed mechanisms in the behavioral finance literature, such as overconfidence highlighted by Daniel, Hirshleifer, and Subrahmanyam (1998), slow information diffusion by Hong and Stein (1999), and extrapolation by Barberis, Shleifer and Vishny (1998) and Barberis et al. (2014).

Our model also differs from Burnside, Eichenbaum, and Rebelo (2013), which offers a model of housing market booms and busts based on the epidemic spreading of optimistic or pessimistic beliefs among home buyers through their social interactions. Our learning-based mechanism is also different from Nathanson and Zwick (2014), which studies the hoarding of land by home builders in certain elastic areas as a mechanism to amplify price volatility in the recent U.S. housing cycle. Informational frictions in our model anchor on the elasticity of housing supply, which is different from the amplification to price volatility induced by dispersed information and short-sale constraints featured in Favara and Song (2014). Furthermore, while our model does not differentiate learning of in-town and out-of-town home buyers, our framework can serve as a basis for future studies of out-of-town speculators, which are shown to be important by a recent study of Chinco and Mayer (2013).

By focusing on information aggregation and learning of symmetrically informed households with dispersed private information, our study differs in emphasis from those that analyze the presence of information asymmetry between buyers and sellers of homes, such as Garmaise and Moskowitz (2004) and Kurlat and Stroebel (2014). Our analysis also features a tractable log-linear equilibrium, and therefore differs from the frameworks of Goldstein, Ozdenoren, and Yuan (2013) and Albagi, Hellwig, and Tsyvinski (2012, 2014), which employ risk-neutral agents with normally-distributed asset fundamentals and position limits to deliver tractable nonlinear equilibria.

There are extensive studies in the housing literature highlighting the roles played by both demand-side and supply-side factors in driving housing cycles. On the demand side, Himmelberg, Mayer and Sinai (2006) focuses on interest rates, Poterba (1991) on tax changes, and Mian and Sufi (2009) on credit expansion. On the supply side, Glaeser, Gyourko, Saiz (2008) emphasizes supply as a key force in mitigating housing bubbles, Haughwout, Peach, Sporn and Tracy (2012) provides a detailed account of the housing supply side during the U.S. housing cycle in the 2000s, and Gyourko (2009) systematically reviews the literature on housing supply. By introducing informational frictions, our analysis shows that supply-side and demand-side factors are not mutually independent. In particular, supply shocks may affect housing demand by acting as informational noise in household learning and thus influencing households' expectations of the strength of the neighborhood.

1 Some Basic Facts

Before we present a model to analyze how supply elasticity affects learning in housing markets, we present some basic facts regarding the relationship between supply elasticity and the magnitudes of housing price booms and busts experienced by different counties during the recent U.S. housing cycle. Even though common wisdom holds that supply elasticity attenuates boom and bust cycles, the data does not support a robust, monotonic relationship between the magnitude of the housing cycle in a county and its supply elasticity. In fact, our analysis uncovers that counties with supply elasticities in an intermediate range had experienced more dramatic housing booms and busts than counties with the most inelastic supply.

Our focus is on the most recent housing cycle, which was a national cycle for the US housing market. Many factors, such as the Clinton-era initiatives to broaden homeownership, the low interest rate environment of the late 1990's and early 2000's, the inflow of foreign capital, and the increase in securitization and sub-prime lending, contributed to the boom. While this was a well-known national phenomenon at the time, how these factors expressed themselves at the regional level was more idiosyncratic and uncertain. The magnitudes of housing price cycle experienced by different regions reflect such idiosyncratic uncertainty, which is the focus of our empirical as well as theoretical analysis.[1]

Our county-level house price data comes from the Case-Shiller home price indices, which are constructed from repeated home sales. There are 420 counties in 46 states with a large enough number of repeat home sales to compute the Case-Shiller home price indices. We use the Consumer Price Index (CPI) from the Bureau of Labor Statistics to deflate the Case-Shiller home price indices. In addition, we also use population data from the 2000 U.S. census.

For housing supply elasticity, we employ the commonly-used elasticity measure constructed by Saiz (2010). This elasticity measure focuses on geographic constraints by defining undevelopable land for construction as terrain with a slope of 15 degrees or more and areas lost to bodies of water including seas, lakes, and wetlands. This measure provides an exogenous measure of supply elasticity, with a higher value if an area is more geographically restricted. Saiz's measure is available for 269 Metropolitan Statistical Areas (MSAs). By matching counties with MSAs, our sample includes 326 counties for which we have data on both house prices and supply elasticity available from 2000 to 2010. Though our sample covers only 11 percent of the counties in the U.S., they represent 53 percent of the U.S. population and 57 percent of the housing trading volume in 2000.

[1] The regional uncertainty introduced by this national phenomenon is absent from the local boom and bust episodes throughout the 1970's and 1980's. While there are other national housing cycles in history, data limitations restrict our attention to the most recent US housing cycle.

Figure 1 displays the real home price indices for the U.S. and three cities, New York, Las Vegas, and Charlotte, from 2000 to 2010. We normalize all indices to 100 in 2000. The national housing market experienced a significant boom and bust cycle in the 2000s with the national home price index increasing over 60 percent from 2000 to 2006 and then falling back to the 2000 level through 2010. Different cities in the U.S experienced largely synchronized price booms and busts during this period, even though the magnitudes of the cycle varied across these cities. According to Saiz's measure, the elasticity measure for New York, Las Vegas, and Charlotte are 0.76, 1.39, and 3.09, respectively. New York, which has severe geographic constraints and building regulations, had a real housing price appreciation of more than 80 percent during the boom, and then declined by over 25 percent during the bust. Charlotte, with its vast developable land and few building restrictions, had an almost flat real housing price level throughout this decade. Sitting in between New York and Charlotte, Las Vegas, with its intermediate supply elasticity, experienced the most pronounced price expansion of over 120 percent during the boom, and the most dramatic price drop of over 50 percent during the bust. Many commentators, including Glaeser (2013), have pointed out that the dramatic boom and bust cycles experienced by Las Vegas and other cities such as Phoenix are peculiar given the relatively elastic supply in these areas.

Are Las Vegas and Phoenix unique in experiencing these dramatic housing cycles despite their relatively elastic housing supply? We now systematically examine this issue by sorting different counties in our sample into three groups, an inelastic, a middle, and an elastic group, based on Saiz's elasticity measure, each with the same number of counties. Figure 2 plots the average price expansion and contraction experienced by each group during the housing cycle (the top panel), together with the total population in each group (the bottom panel). We measure the price expansion in 2004-2006, the period that is often defined as the housing bubble period, and the price contraction in 2007-2009.[2]

The top panel of Figure 2 shows that the inelastic group had the largest house price expansion in 2004-2006 and the largest price contraction in 2007-2009, the middle group experienced a milder cycle, and the elastic group had the most modest cycle. This pattern appears to be consistent with the aforementioned common wisdom that supply elasticity attenuates housing cycles.

It seems natural to sort the counties into several groups each with an equal number of

[2] We have also used an alternative boom period of 2001-2006 and obtained qualitatively similar results as defining the boom in 2004-2006.

counties. In fact, this is a common practice used in the literature to demonstrate a monotonic relationship between housing cycles and supply elasticity. Interestingly, the bottom panel of Figure 2 shows that the population is unevenly distributed across the three groups, with the inelastic group having more than half of the total population. This is consistent with the fact that inelastic areas tend to be densely populated. As the inelastic group pools together a large fraction of the population, there might be substantial heterogeneity between counties within the inelastic group. Indeed, both New York and Las Vegas fall into this inelastic group. This consideration motivates us to examine alternative ways of grouping the counties.

In Figure 3, we sort the counties into ten groups from the most inelastic group to the most elastic group, still with each group holding an equal number of counties. The top panel shows that the housing price expansion and contraction experienced by these ten groups are no longer monotonic with elasticity. In particular, group 3, which has the third most inelastic supply, experienced the largest price expansion during the boom, and the largest price contraction during the bust. Interestingly, Las Vegas falls into group 3, while New York into group 1. The bottom panel again shows that the population tends to be concentrated in the more inelastic groups. Taken together, Figure 3 shows that the commonly perceived, monotonic relationship between housing cycles and supply elasticity is not robust.

In Figure 4, we sort the counties into three groups based on supply elasticity in an alternative way. Instead of letting different groups have an equal number of counties, we let them have the same population. If the magnitude of the housing cycle is monotonically decreasing with supply elasticity, whether we group the counties by number or population should not affect the monotonically decreasing pattern across the groups. In contrast, the top panel of Figure 4 shows that the middle group has the most pronounced housing cycle, with its price expansion during the boom being substantially more pronounced than that of the inelastic group, and its price contraction during the bust slightly greater than that of the inelastic group. The bottom panel shows that the inelastic group has only 40 counties, the middle group slightly below 120 counties, and the elastic group over 160 counties. Under this grouping, while New York remains in the inelastic group, Las Vegas is now in the middle group.

In Figure 5, we further sort the counties into ten groups from the most inelastic group to the most elastic group, with each group having the same population. This figure shows a finer non-monotonicity with groups 3 and 5 experiencing the most pronounced price expansions and contractions.

To further examine whether the more pronounced housing cycles experienced by the intermediate elasticity groups are robust to controlling for other fundamental factors, such as changes of income and population and fraction of subprime households, we adopt a regression approach. Specifically, we separately regress the housing price expansion in 2004-2006 and contraction in 2007-2009 on two dummy variables that indicate whether a county is in the middle elasticity group or the most elastic group, which are constructed in Figure 4, together with a list of control variables. This regression implicitly uses the inelastic group as the benchmark for the middle and elastic groups. The control variables include the fraction of subprime households in the county in 2005, which is computed based on individual mortgage loan applications reported by the "Home Mortgage Disclosure Act" (HMDA) dataset, as well as the contemporaneous population change and annualized per capita income change.

Table 1 reports the regression results. Columns 1 and 2 report the regressions of the housing price expansion in 2004-2006 without and with the controls. Columns 3 and 4 report the regressions of the housing price contraction in 2007-2009 without and with the controls. Among the control variables, the fraction of subprime households is significantly correlated with both the price expansion during the boom and the price contraction during the bust. This result is consistent with Mian and Sufi (2009), which shows that credit expansion to subprime households before 2006 was a key factor in explaining the recent housing cycle. The changes in population and income are insignificant in explaining either the price expansion or the contraction across the cycle. More importantly, even after controlling for these fundamental factors, the middle group experienced a significantly more pronounced housing price expansion in 2004-2006 and a more pronounced price contraction in 2007-2009 relative to the inelastic group.

It is important to note that the findings shown in Figures 2-5 and Table 1 are not driven by a few areas such as Las Vegas and Phoenix. In unreported analysis, we have dropped Las Vegas and Phoenix and found that results are similar both qualitatively and quantitatively. These robustness results are available upon request.

Taken together, Figures 2-5 and Table 1 show that the commonly perceived, monotonic relationship between housing cycle and supply elasticity is not robust to finer groupings of counties. Finer groupings and an alternative method of grouping counties by population rather than county number, however, reveal a robust non-monotonic relationship in which the counties in a median elasticity range experienced more pronounced price booms and busts in the 2000s than counties with the most inelastic supply. This non-monotonic relationship is intriguing and cannot easily be explained by the usual role of elasticity in affecting the supply side of housing. In the next section, we present a simple model to illustrate a learning mechanism through which supply elasticity affects the informational role of housing prices and households' learning from housing prices.

2 A Baseline Model

In this section we develop a simple model with two dates $t = 1, 2$ to analyze the effects of informational frictions on the housing market equilibrium in a closed neighborhood. One can think of this neighborhood as a county or a township. A key feature of the model is that the housing market is not only a place for households to trade housing but also a platform to aggregate private information about the unobservable strength or quality of the neighborhood. In addition to its direct role in affecting housing supply in the neighborhood, supply elasticity also indirectly affects the informational noise in the housing prices.

2.1 Model setting

There are two types of agents in the economy: households looking to buy homes in the neighborhood and home builders. Suppose that the neighborhood is new and all households purchase houses from home builders in a centralized market at $t = 1$ and consume both housing and consumption goods at $t = 2$.[3]

Each household cares about the strength of the neighborhood, as its utility depends on not only its own housing consumption but also the housing consumption of other households in their neighborhood. This assumption is motivated by the empirical findings of Ioannides and Zabel (2003) and leads to strategic complementarity in each household's housing demand.[4] The strength of this closed neighborhood is reflected by the aggregate productivity of its households. A strong aggregate productivity implies greater output by all households, and thus greater housing demand by them as well. In the presence of realistic informa-

[3] For simplicity, we do not consider the endogenous decision of households choosing their neighborhood, and instead take the pool of households in the neighborhood as given. See Van Nieuwerburgh and Weill (2010) for a systematic treatment of moving decisions by households across neighborhoods.

[4] There are other types of social interactions between households living in a neighborhood, which are explored, for instance, in Durlauf (2004) and Glaeser, Sacerdote, and Scheinkman (2003).

tional frictions in gauging the strength of the neighborhood, the housing market provides an important platform for aggregating information about this aggregate productivity. As a consequence, the resulting housing price serves as a useful signal about the neighborhood's strength.

2.1.1 Demand side

There is a continuum of households, indexed by $i \in [0,1]$. Household i has a Cobb-Douglas utility function over its own housing H_i, consumption good C_i, and the housing consumption of all other households in the neighborhood $\{H_j\}_{j\in[0,1]}$:[5]

$$U\left(\{H_j\}_{j\in[0,1]}, C_i\right) = \left\{\frac{1}{1-\eta_H}\left(\frac{H_i}{1-\eta_c}\right)^{1-\eta_c}\left(\frac{\int_{[0,1]/i} H_j dj}{\eta_c}\right)^{\eta_c}\right\}^{1-\eta_H}\left(\frac{1}{\eta_H}C_i\right)^{\eta_H}. \quad (1)$$

The parameters $\eta_H \in (0,1)$ and $\eta_c \in (0,1)$ measure the weights of different consumption components in the utility function. A higher η_H means a stronger complementarity between housing consumption and goods consumption, while a higher η_c means a stronger complementarity between the housing of household i and housing of the composite house $\int_{[0,1]/i} H_j dj$ purchased by the other households in the neighborhood.

The production function of household i is $e^{A_i} l_i$, where l_i is the household's labor choice and A_i is its productivity. A_i is composed of a component A common to all households in the neighborhood and an idiosyncratic component ε_i:

$$A_i = A + \varepsilon_i,$$

where $A \sim \mathcal{N}\left(\bar{A}, \tau_A^{-1}\right)$ and $\varepsilon_i \sim \mathcal{N}\left(0, \tau_\varepsilon^{-1}\right)$ are both normally distributed. The common productivity A represents the strength of the neighborhood, as a higher A implies a more productive neighborhood. As A determines the households' aggregate demand for housing, it represents the demand-side fundamental.

As a result of realistic informational frictions, neither A nor A_i is observable to the households. Instead, each household observes a noisy private signal about A at $t = 1$. Specifically, household i observes

$$\theta_i = A + \nu_i,$$

[5] Our modeling choice of non-separable preferences for housing and consumption is similar to the CES specification of Piazzesi, Schneider, and Tuzel (2007).

where $\nu_i \sim \mathcal{N}\left(0, \tau_\theta^{-1}\right)$ is signal noise independent across households. The parameter τ_θ measures the precision of the private signal. As $\tau_\theta \to \infty$, the households' signals become infinitely precise and the informational frictions about A vanish.

While our model setting is static and focuses on a closed neighborhood, one can provide a broad interpretation of the uncertainly in the neighborhood strength A. In relating this setting to the recent housing cycle, we interpret the uncertainty in A as being induced by a nation-wide shock to credit expansion for homeowners in the 2000's due to the factors mentioned in Section 1. While this shock affected the whole country, its potential impact on individual neighborhoods was different. Some neighborhoods might attract migrants with high productivity from other neighborhoods as a result of the national shock, while others might lose their high-quality residents who could now more easily re-allocate to other locations. As a result, home buyers faced a realistic problem in inferring how this national shock might have influenced the strength of an individual neighborhood when buying a home.

Households care about the strength of the neighborhood A not only because it determines their own productivity, but also because of complementarity in their housing demand. Since households want to live in similar-sized houses to their neighbors, they need to learn about A because it affects their neighbors' housing decisions. Consequently, while a household may have a fairly good understanding of its own productivity when moving into a neighborhood, complementarity in housing demand motivates it to pay attention to housing prices to learn about the average level A for the neighborhood.

We assume that each household experiences a disutility for labor $\frac{l_i^{1+\psi}}{1+\psi}$, and that it maximizes its expected utility at $t = 1$ by choosing its housing demand H_i and labor l_i:

$$\max_{\{H_i, l_i\}} E\left[U\left(\{H_j\}_{j\in[0,1]}, C_i\right) - \frac{l_i^{1+\psi}}{1+\psi}\middle|\mathcal{I}_i\right] \tag{2}$$

$$\text{such that} \quad C_i = e^{A_i} l_i - PH_i + \Pi_i.$$

We assume for simplicity that the home builder for household i is part of the household, and that the builder brings home its profit $\Pi_i = PH_i$ to the household after construction has taken place. Furthermore, we normalize the interest rate from $t = 1$ to $t = 2$ to be zero. As a result, at $t = 2$, household i's budget constraint satisfies $C_i = e^{A_i} l_i$. The choices of labor and housing are made at $t = 1$ subject to each household's information set $\mathcal{I}_i = \{\theta_i, P\}$, which includes its private signal θ_i and the housing price P.[6]

2.1.2 Supply side

Home builders face a convex labor cost

$$\frac{k}{1+k} e^{-\zeta} S_H^{\frac{1+k}{k}}$$

in supplying housing, where S_H is the quantity of housing supplied, $k \in (0, \infty)$ is a constant parameter, and ζ represents a shock to the building cost. We assume that ζ is observed by builders but not households,[7] and that from the perspective of households $\zeta \sim \mathcal{N}\left(\bar{\zeta}, 1\right)$, i.e., a normal distribution with $\bar{\zeta}$ as the mean and unit variance.

Builders at $t = 1$ maximize their profit subject to their supply curve

$$\Pi\left(S_H\right) = \max_{S_H}\ PS_H - \frac{k}{1+k} e^{-\zeta} S_H^{\frac{1+k}{k}}. \tag{3}$$

It is easy to determine the builders' optimal supply curve:

$$S_H = P^k e^{\xi}, \tag{4}$$

where $\xi = k\zeta$ has the interpretation of being a supply shock with normal distribution $\xi \sim \mathcal{N}\left(\bar{\xi}, k^2\right)$, where $\bar{\xi} = k\bar{\zeta}$. The parameter k measures the supply elasticity of the neighbohood. A more elastic neighborhood has a larger supply shock, i.e., the supply shock has greater mean and variance. In the housing market equilibrium, the supply shock ξ not only affects the supply side, but also the demand side, as it acts as informational noise in the price signal when the households use the price to learn about the common productivity A.

We also incorporate a behavioral feature that households may underestimate the supply elasticity in the neighborhood, and incorrectly believe it to be ϕk rather than k, where $\phi \leq 1$. This feature is motivated by the observation made by Glaeser (2013) that agents tend to underestimate supply shocks during various episodes of real-estate speculation observed in U.S. history. Specifically, Glaeser identifies the under-appreciation of the supply response

[6] We do not include the volume of housing transactions in the information set as a result of a realistic consideration that, in practice, people observe only delayed reports of total housing transactions at highly aggregated levels, such as national or metropolitan levels.

[7] Even though we assume that builders perfectly observe the supply shock, a more realistic setting would have builders each observing part of the supply and thus needing to aggregate their respective information in order to fully observe the supply-side shock. We have explored this more general setting, which entails an additional layer of information aggregation on the builder side of the housing market. Nevertheless, it gives qualitatively similar insights as our current setting.

by buyers as a systematic, cognitive limitation that helps explain historical boom and bust episodes of real-estate speculation. As a result of this behavioral feature, households may put too much weight on housing prices in their housing decisions because they overestimate the precision of prices as a signal about the neighborhood strength A.

This overweighing is reminiscent of extrapolative beliefs, which, as referenced in the introduction, have been recognized by the literature as important for understanding housing cycles. It is useful to note that while extrapolative beliefs amplify housing price volatility, they nevertheless imply that price volatility is monotonically decreasing in supply elasticity. The overweighing of prices highlighted in our model anchors on a characteristic of neighborhoods that allows for it to explain the hump-shaped pattern in price volatility across supply elasticity.

2.2 The equilibrium

Our model features a noisy rational expectations equilibrium, which requires clearing of the housing market that is consistent with the optimal behavior of both households and home builders:

- Household optimization: $\left\{ \{H_i\}_{i\in[0,1]}, l_i \right\}$ solves each household's maximization problem in (2).
- Builder optimization: S_H solves the builders' maximization problem in (3).
- At $t = 1$, the housing market clears:

$$\int_{-\infty}^{\infty} H_i\left(\theta_i, P\right) d\Phi\left(\nu_i\right) = P^k e^{\xi},$$

  where each household's housing demand $H_i\left(\theta_i, P\right)$ depends on its private signal θ_i and the housing price P. The demand from households is integrated over the idiosyncratic component of their private signals $\{\nu_i\}_{i\in[0,1]}$.

We first solve for the optimal labor and housing choices for a household given its utility function and budget constraint in (2), which are characterized in the following proposition.

Proposition 1 *Households i's optimal labor choice depends on its expected productivity:*

$$l_i = \left\{ \eta_H E\left[\left\{ \frac{1}{1-\eta_H} \left(\frac{H_i}{1-\eta_c} \right)^{1-\eta_c} \left(\frac{\int_{[0,1]/i} H_j dj}{\eta_c} \right)^{\eta_c} \right\}^{1-\eta_H} \left(\frac{1}{\eta_H} e^{A_i} \right)^{\eta_H} \middle| \mathcal{I}_i \right] \right\}^{\frac{1}{1+\psi-\eta_H}},$$

and its demand for housing is

$$\begin{aligned}\log H_i &= \frac{2+\psi-\eta_H}{\psi+(1-\eta_H)\eta_c}\log E\left[\left(\int_{[0,1]} H_j dj\right)^{\eta_c(1-\eta_H)} e^{\eta_H A_i}\middle|\mathcal{I}_i\right] - \frac{1+\psi-\eta_H}{\psi+(1-\eta_H)\eta_c}\log P \\ &-\frac{1+\psi-\eta_H}{\psi+(1-\eta_H)\eta_c}\log E\left[\left(\int_{[0,1]} H_j dj\right)^{\eta_c(1-\eta_H)} e^{(\eta_H-1)A_i}\middle|\mathcal{I}_i\right] \\ &+\frac{\psi}{\psi+(1-\eta_H)\eta_c}\log\left((1-\eta_c)\left(\frac{1-\eta_H}{\eta_H}\right)\left(\frac{1-\eta_c}{\eta_c}\right)^{\frac{\eta_c(1-\eta_H)}{\psi}}\right). \qquad (5)\end{aligned}$$

Proposition 1 demonstrates that the labor chosen by a household is determined by its expected productivity, and that its housing demand is determined by not only its own productivity e^{A_i} but also the aggregate housing consumption of other households. This latter component arises from the complementarity in the utility function of the household.

By clearing the aggregate housing demand of the households with the supply from the builders, we derive the housing market equilibrium. Despite the nonlinearity in each household's demand and in the supply from builders, we obtain a tractable unique log-linear equilibrium. The following proposition summarizes the housing price and each household's housing demand in this equilibrium.

Proposition 2 *At $t=1$, the housing market has a unique log-linear equilibrium: 1) The housing price is a log-linear function of A and ξ:*

$$\log P = p_A A + p_\xi \xi + p_0, \qquad (6)$$

with the coefficients p_A and p_ξ given by

$$\begin{aligned} p_A &= \frac{1+\psi}{1+\psi(1+k)-\eta_H} - \frac{\psi+(1-\eta_H)\eta_c}{1+\psi(1+k)-\eta_H}\tau_\theta^{-1}\tau_A b > 0, \qquad (7)\\ p_\xi &= -\frac{\psi}{1+\psi(1+k)-\eta_H} - \frac{\psi+(1-\eta_H)\eta_c}{1+\psi(1+k)-\eta_H}\tau_\theta^{-1}\left(\frac{b}{\phi k}\right)^2 < 0, \qquad (8)\end{aligned}$$

where $b \in \left[0, \frac{1+\psi}{\psi}\frac{\tau_\theta}{\left(1+\frac{(1-\eta_H)\eta_c}{\psi}\right)\tau_A+\tau_\theta}\right]$ is the unique positive, real root of equation (29), and p_0 is given in equation (34).

2) The housing demand of household i is a log-linear function of its private signal θ_i and $\log P$:

$$\log H_i = h_\theta \theta_i + h_P \log P + h_0, \qquad (9)$$

with the coefficients h_θ and h_P given by

$$h_\theta = b > 0, \tag{10}$$

$$h_P = -\frac{1+\psi-\eta_H}{\psi} + \frac{1+\psi+\eta_c(1-\eta_H)b}{\psi} \frac{\left(\frac{b}{\phi k}\right)^2}{\tau_A + \tau_\theta + \left(\frac{b}{\phi k}\right)^2} \frac{1}{p_A}, \tag{11}$$

and h_0 given by equation (23).

Proposition 2 establishes that the housing price P is a log-linear function of the neighborhood strength A and the housing supply shock ξ, and that each household's housing demand is a log-linear function of its private signal θ_i and the log housing price $\log P$. Similar to Hellwig (1980), the housing price aggregates the households' dispersed private information to partially reveal A. The price does not depend on the idiosyncratic noise in any individual household's signal because of the Law of Large Numbers. This last observation is key to the tractability of our model, and ensures that the housing demand from the households retains a log-normal distribution after aggregation.

In the presence of informational frictions, the housing supply shock ξ serves the same role as noise trading in standard models of asset market trading with dispersed information. This feature is new to the housing literature and highlights an important channel for supply shocks to affect the expectations of potential home buyers. Since households cannot perfectly disentangle changes in housing prices caused by supply shocks from those brought about by shocks to demand, they partially confuse a housing price change caused by a supply shock to be a signal about the strength of the neighborhood.

To facilitate our discussion of the impact of learning, it will be useful to introduce a perfect-information benchmark in which all households perfectly observe the strength of the neighborhood A. The following proposition characterizes this benchmark equilibrium.

Proposition 3 *Consider a benchmark setting, in which households perfectly observe A (i.e., $\theta_i = A$, $\forall i$.) There is also a log-linear equilibrium, in which the housing price is*

$$\log P = \frac{1+\psi}{1+\psi(1+k)-\eta_H} A - \frac{\psi}{1+\psi(1+k)-\eta_H}\xi + \frac{(1+\psi)\eta_H - (1-\eta_H)(1+\psi-\eta_H)}{2(1+\psi(1+k)-\eta_H)}\tau_\varepsilon^{-1}$$
$$+\frac{\psi}{1+\psi(1+k)-\eta_H}\log\left((1-\eta_c)\left(\frac{1-\eta_H}{\eta_H}\right)\left(\frac{1-\eta_c}{\eta_c}\right)^{\frac{\eta_c(1-\eta_H)}{\psi}}\right).$$

and all households have the same housing demand

$$\log H = \frac{1+\psi}{\psi}A - \frac{1+\psi-\eta_H}{\psi}\log P + \frac{(1+\psi)\eta_H - (1-\eta_H)(1+\psi-\eta_H)}{2\psi}\tau_\varepsilon^{-1} + \log\left((1-\eta_c)\left(\frac{1-\eta_H}{\eta_H}\right)\left(\frac{1-\eta_c}{\eta_c}\right)^{\frac{\eta_c(1-\eta_H)}{\psi}}\right).$$

Furthermore, the housing market equilibrium with information frictions characterized in Proposition 2 converges to this benchmark equilibrium as $\tau_\theta \nearrow \infty$, *and the variance of the housing price* $Var[\log P]$ *has a U-shaped relationship with the supply elasticity* k.

It is reassuring that as the households' private information becomes infinitely precise, the housing market equilibrium converges to the perfect-information benchmark. In this perfect-information benchmark, the housing price is also a log-linear function of the demand-side fundamental A and the supply shock ξ, and each household's identical demand is a log-linear function of the perfectly observed A and the housing price $\log P$. Consistent with the standard intuition, a higher A increases both the housing price and aggregate housing demand, while a larger supply shock ξ reduces the housing price but increases aggregate housing demand. It is also easy to see that in this benchmark setting, as the supply elasticity k rises from zero to infinity, the weight of A (the demand-side fundamental) in the housing price decreases, while the weight of ξ (the supply-side shock) increases.

Furthermore, in the perfect-information benchmark, the housing price variance has a U-shaped relationship with the housing supply elasticity k. This is because, as k varies, it causes the housing price to assign different weights to the demand-side fundamental and the supply-side shock. The standard intuition from diversification implies that the price has the lowest variance when the weights of the two factors are balanced, i.e., the supply elasticity takes an intermediate value. This U-shaped price variance serves a benchmark to evaluate the housing price variance in the presence of informational frictions.

2.3 Impact of learning

In the presence of informational frictions about the strength of the neighborhood A, each household needs to use its private signal θ_i and the publicly-observed housing price $\log P$ to learn about A. As the housing price $\log P$ is a linear combination of the demand-side fundamental A and the housing supply shock ξ, the supply shock interferes with this learning process. A larger supply shock ξ, by depressing the housing price, will have an additional effect of reducing the households' expectations of A. This, in turn, reduces their housing demand and consequently further depresses the housing price. This learning effect thus causes the supply shock to have a larger negative effect on the equilibrium housing price than it would in the perfect-information benchmark. Similarly, this learning effect also causes the demand-side fundamental A to have a smaller positive effect on the price than in the perfect-information benchmark because informational frictions cause households to partially discount the value of A. The following proposition formally establishes this learning effect on the housing price.

Proposition 4 *In the presence of informational frictions, coefficients* $p_A > 0$ *and* $p_\xi < 0$ *derived in Proposition 2 are both lower than their corresponding values in the perfect-information benchmark.*

The precision of the households' private information τ_θ determines the informational frictions they face. The next proposition establishes that an increase in τ_θ mitigates the informational frictions and brings the coefficient p_A closer to its value in the perfect-information benchmark. In fact, as τ_θ goes to infinity, the housing market equilibrium converges to the perfect-information benchmark (Proposition 3).

Proposition 5 p_A *is monotonically increasing with the precision* τ_θ *of each household's private signal and decreasing with the degree of complementarity in households' housing consumption* η_c.

Each household's housing demand also reveals how the households learn from the housing price. In the presence of informational frictions about A, the housing price is not only the cost of acquiring shelter but also a signal about A. The housing demand of each household derived in (9) reflects both of these effects. Specifically, we can decompose the price elasticity of each household's housing demand h_P in equation (11) into two components: The first component $-\frac{1}{\eta_H}$ is negative and represents the standard cost effect (i.e., downward sloping demand curve), as in the perfect-information benchmark in Proposition 3, and the second component $\frac{1+\psi+\eta_c(1-\eta_H)b}{\psi}\frac{\left(\frac{b}{\phi k}\right)^2}{\tau_A+\tau_\theta+\left(\frac{b}{\phi k}\right)^2}\frac{1}{p_A}$ is positive and represents the learning effect.

A higher housing price raises the household's expectation of A and induces it to consume more housing through two related but distinct learning channels. First, a higher A implies a higher productivity for the household itself. Second, a higher A also implies that other households demand more housing, which in turn induces each household to demand more housing.

As a reflection of this complementarity effect, the second component in the price elasticity of housing demand increases with η_c, the degree of complementarity in the household's utility of its own housing consumption and other households' housing consumption.

As a result of the presence of the complementarity channel, η_c also affects the impact of learning on the housing price. As η_c increases, each household puts a greater weight on the housing price in its learning of A and a smaller weight on its own private signal. This in turn makes the housing price less informative of A. In this way, a larger η_c exacerbates the informational frictions faced by households. Indeed, Proposition 5 shows that the loading of $\log P$ on A is decreasing with η_c.

Housing supply elasticity k plays an important role in determining the informational frictions faced by the households, in addition to its standard supply effect. To illustrate this learning effect of supply elasticity, we consider two limiting economies as k goes to 0 and ∞, which are characterized in the following proposition.

Proposition 6 *As $k \to \infty$, the housing price and each household's housing demand converge to*

$$\log P = -\zeta,$$

and

$$\log H_i = \frac{1+\psi}{\psi} \frac{\tau_\theta}{\tau_A + \tau_\theta + \frac{(1-\eta_H)\eta_c}{\psi}\tau_A}\theta_i - \frac{1+\psi-\eta_H}{\psi}\log P + h_0.$$

As $k \to 0$, the housing price and each household's housing demand converge to

$$\begin{aligned}\log P \quad &= \quad \frac{1+\psi}{1+\psi-\eta_H}A + \frac{1}{2}\frac{(1+\psi)\,\eta_H - (1-\eta_H)\,(1+\psi-\eta_H)}{1+\psi-\eta_H}\tau_\varepsilon^{-1} \\ &\quad + \frac{\psi}{1+\psi-\eta_H}\log\left((1-\eta_c)\left(\frac{1-\eta_H}{\eta_H}\right)\left(\frac{1-\eta_c}{\eta_c}\right)^{\frac{\eta_c(1-\eta_H)}{\psi}}\right),\end{aligned}$$

and $\log H_i = 0$.

At one end, as supply elasticity goes to zero, the housing price is completely driven by A and thus fully reveals it. In this case, each household precisely learns A from the price, and as a result, both the housing price and each household's housing demand coincide with their corresponding values in the prefect-information benchmark. At the other end, as supply elasticity goes to infinity, the housing price is completely driven by the supply shock ξ and contains no information about A. In this case, each household has to rely on its own private signal to infer A. As the housing price, however, is fully determined by the supply shock and independent of the demand-side fundamental, informational frictions about A do not matter for the housing price. Consequently, the housing price also coincides with that in the perfect-information benchmark, even though informational frictions still affect each household's housing demand. Taken together, when housing supply is either perfectly elastic or inelastic, the housing price is not affected by informational frictions and coincides with that in the perfect-information benchmark.

The following proposition characterizes the housing price at an intermediate supply elasticity and, in particular, analyzes the role of the households' underestimation ϕ of supply elasticity.

Proposition 7 *Consider an intermediate level of supply elasticity* $k \in (0, \infty)$. *1) In the presence of informational frictions, both* p_A *and* $|p_\xi|$ *are monotonically decreasing with* ϕ. *2) When* $\phi = 1$, *the housing price variance with informational frictions is lower than that of the perfect-information benchmark. 3) The variance of the housing price* $\log P$ *is monotonically decreasing with* ϕ, *and a sufficient condition* $1 - \frac{\psi}{\psi+(1-\eta_H)\eta_c}\frac{\tau_\theta}{\tau_A} \leq \phi^2 \leq \frac{1}{2}$ *ensures the price variance to be at least as large as its corresponding value in the perfect-information benchmark.*

Proposition 7 shows that, at an intermediate supply elasticity, the households' underestimation of supply elasticity causes them to over-interpret the information contained in the price signal and thus overreact to the price signal. Consequently, the positive loading of the equilibrium housing price p_A on the demand-side fundamental A becomes larger and the negative loading p_ξ on the supply shock becomes more negative. That is, the housing price becomes more responsive to both demand and supply shocks.

Proposition 7 also shows that, in the absence of the households' underestimation of supply elasticity, the presence of informational frictions reduces the housing price variance. This is because informational frictions make households less responsive to demand shocks, causing the housing price to load less on demand shocks. When households underestimate the supply elasticity ($\phi < 1$), their overreaction to the price signal amplifies the price effects of both supply and demand shocks, and implies that the housing price variance is monotonically decreasing with ϕ. In fact, Proposition 7 shows that when ϕ is sufficiently small, the housing price variance is at least as large as its value in the perfect-information benchmark.

Interestingly, the volatility amplification induced by the households' overreaction to the housing price is most pronounced when the supply elasticity is in an intermediate range. This follows from our earlier discussion of the two limiting cases when the elasticity goes to either zero or infinity. At one end, when the supply is infinitely elastic, the households' learning about the demand side is irrelevant for the price. At the other end, when the supply is infinitely inelastic, the price fully reveals the demand-side fundamental and there is no room for the households to overreact. In between these two limiting cases, the demand-side fundamental plays a significant role in determining the housing price and at the same time households face substantial uncertainty about the demand-side fundamental, which leaves room for their overreaction to amplify the price volatility.

In Figure 6, we provide a numerical example to illustrate how informational frictions and households' overreaction jointly affect the housing price variance. The figure depicts the log-price variance $Var[\log P]$ against the supply elasticity under the following parameter values:

$$\tau_\theta = 0.1,\ \tau_A = 1, \phi = 0.1,\ \psi = 0.6, \eta_c = 0.5, \eta_H = 0.9.$$

For comparison, it also depicts the log-price variance in the perfect-information benchmark, which is obtained as $\tau_\theta \to \infty$. As the supply elasticity k rises from 0 to 1 (i.e., from infinitely inelastic to more elastic), the log-price variance decreases with the supply elasticity. In contrast, when the households face informational frictions with $\tau_\theta = 1$, Figure 6 shows that the log-price variance first increases with k, when k is lower than an intermediate level around 0.1, and then decreases with k.[8] The difference between this humped shape and the monotonically decreasing curve in the perfect-information benchmark illustrates the joint effect of informational frictions and the households' overreaction to the price signal.

The humped log-price variance illustrated in Figure 6 provides an explanation for the aforementioned, non-monotonic relationship between the housing boom and bust cycles experienced by different U.S. counties in the 2000s and supply elasticity.

3 Elasticity and Housing Speculation

In this section, we further explore the effects of household learning on housing speculation. We first extend the baseline model presented in the last section to incorporate secondary

[8] Outside the range of k depicted in the figure, both of these two lines are decreasing and eventually converge to each other as $k \to \infty$, as derived in Proposition 6.

homes and, in particular, to show that the same learning effect discussed earlier leads to new predictions regarding the relationship between housing speculation and supply elasticity. Then, we examine these predictions in the data and provide some supportive evidence.

3.1 A model extension

We extend the model presented in the previous section to incorporate three types of agents in the economy: households, home builders, and immigrants looking to move into the neighborhood. These immigrants are the new addition to this extension. Suppose that these immigrants make their decision at $t = 1$ on whether to move into the neighborhood, based on the expected strength of the neighborhood. The immigrants arrive in the neighborhood at $t = 2$ and then buy the secondary homes initially owned by the households.

Households At $t = 1$, households purchase two types of homes, one as their primary residence and the other as a secondary home to sell at $t = 2$ to the immigrants. Home builders build and sell these two types of homes in two separate housing markets. This separate treatment of primary and secondary homes is consistent with the fact that, in practice, primary homes tend to be single houses, while secondary homes tend to be apartments and condominiums. Another advantage of giving separate supply curves to primary and secondary homes is that it ensures a tractable log-linear equilibrium.

When making their decisions at $t = 1$, households again receive a private signal θ_i about the strength of the neighborhood. Like before, the demand of household i for primary home is H_i, but now, in addition, the household has a demand for secondary home M_i. For simplicity, suppose that households have no initial wealth and must finance their purchases by borrowing debt D_i from home builders.[9] We also normalize the interest rate on the loans to be zero. Then, the budget constraint of household i at $t = 1$ is

$$PH_i + Q_1 M_i = D_i, \tag{12}$$

where P is the price of primary homes and Q_1 is the price of secondary homes.

At $t = 2$, households decide how much of their goods to produce, sell their secondary homes at a price Q_2 to the immigrants that have moved into the neighborhood, and repay their debt to home builders. Household i again employs its own labor l_i as an input to

[9] This assumption is innocuous as our main interest is not to study the effects of the households' credit constraints.

production with production function $e^{A_i} l_i$. As in the baseline model, household i earns the income Π_{Hi} from the home builder of their primary home, who is part of the household. Thus, in equilibrium, $\Pi_{Hi} = PH_i$.

The budget constraint of household i at $t = 2$ is then

$$C_i = -D_i + e^{A_i} l_i + Q_2 M_i + \Pi_{Hi}, \tag{13}$$

where C_i is the goods consumption of household i. Households have the same Cobb-Douglas preferences as in the baseline model for consuming their primary residence and non-housing consumption at $t = 2$.

At $t = 1$, each household maximizes

$$\max_{\{H_i, M_i, l_i\}} E\left[U\left(\{H_j\}_{j\in[0,1]}, C_i \right) - \frac{l_i^{1+\psi}}{1+\psi} \middle| \mathcal{I}_i \right] \tag{14}$$

under its information set $\mathcal{I}_i = \{\theta_i, P, Q_1\}$, which includes its private signal θ_i and the housing prices P and Q_1, and subject to its budget in (12). The household's consumption C_i is determined by its budget constraint at $t = 2$ given in (13).

Home builders Home builders face separate production processes for building primary and secondary homes. Specifically, they face the following convex labor cost for building each type of homes:

$$\frac{k}{1+k} e^{-\zeta_j} S_j^{\frac{1+k}{k}}$$

where $j \in \{H, M\}$ indicates the type of homes with H representing primary homes and M representing secondary homes, S_j is the quantity of type-j homes supplied, and ζ_j represents a supply shock. We assume that ζ_j is observed by builders but not households. From the perspective of households, there are two components in the supply shock of type-j homes:

$$\zeta_j = \zeta + e_j, \ j \in \{H, M\}.$$

The first component ζ is common to the two types of homes. It has a normal distribution with $\bar{\zeta}$ as the mean and unit variance. The second component e_j is idiosyncratic to type-j homes. It has a normal distribution with zero mean and α as its standard deviation.

The builders' optimization determines their supply curves for primary and secondary homes:

$$S_H = P^k e^{\xi_H} \text{ and } S_M = Q_1^k e^{\xi_M},$$

where, for $j \in \{H, M\}$, $\xi_j \sim N\left(\bar{\xi}, (1+\alpha^2) k^2\right)$ and $\bar{\xi} = k\bar{\zeta}$.

Immigrants Immigrants decide on whether they want to move into the neighborhood at $t = 1$, although they move into the neighborhood only at $t = 2$ and thus purchase secondary homes from the initial households at $t = 2$. This is realistic as it takes time for immigrants to move families from one area to another. A key assumption is that immigrants have to make their migration decisions based on their expectations of the strength of a neighborhood at $t = 1$, rather than the realized strength at $t = 2$. As immigrants are from outside of the neighborhood, they do not receive any private information and have to rely on $\mathcal{I}_c = \{P, Q_1\}$, which contains the publicly observable housing prices at $t = 1$, to form their expectations. Like the households, the immigrants also under-estimate the housing supply elasticity by the same factor $\phi \leq 1$.

Anticipating the arrival of immigrants at $t = 2$, the initial households act as intermediaries by buying secondary homes at $t = 1$ and selling them to immigrants at $t = 2$.

It is intuitive that when immigrants hold a higher expectation about the strength of a neighborhood, the neighborhood will attract a larger number of immigrants. This is because immigrants are also more productive in a stronger neighborhood. Consequently, there will be a greater demand for secondary homes at $t = 2$. For simplicity, we adopt a reduced form to capture the immigrants' housing demand by assuming that the aggregate wealth W they bring to buy homes is

$$W = E\left[e^{A} \middle| \mathcal{I}_c \right]. \tag{15}$$

This form allows us to maintain the tractable log-linear equilibrium.[10]

Equilibrium We derive the noisy rational expectations equilibrium as in the baseline model. The equilibrium features the clearing of both primary and secondary homes $t = 1$:

$$\begin{aligned} \int_{-\infty}^{\infty} H_i\left(\theta_i, P, Q_1\right) d\Phi\left(v_i\right) &= P^k e^{\xi_H}, \\ \int_{-\infty}^{\infty} M_i\left(\theta_i, P, Q_1\right) d\Phi\left(v_i\right) &= Q_1^k e^{\xi_M}, \end{aligned}$$

and the households' learning from the prices of both primary and secondary homes. We also require market clearing in the market for secondary homes at $t = 2$, which imposing the

[10] One may micro-found this form in different ways. One possibility is that the number of immigrants increases with their expectation of the strength of neighborhood and each immigrant arrives with a fixed amount of wealth to acquire housing. Another possibility, which we have explicitly worked out in an earlier draft, is to let the immigrants supply labor to the initial households based on their expectations of the strength of the neighborhood, which determines the productivity of the households.

immigrants to spend all their wealth W to purchase secondary homes:

$$Q_2 \int_{-\infty}^{\infty} M_i\left(\theta_i, P, Q_1\right) d\Phi\left(v_i\right) = W.$$

As the nature of the equilibrium and the key steps of deriving the equilibrium are similar to the baseline model, we leave the detailed description and derivation of the equilibrium in an Internet Appendix. Instead, we briefly summarize the key features of the extended model here.

There is a unique log-linear equilibrium where the primary home price is a log-linear function of A, ξ_H, and $\log Q_1$:

$$\log P = p_A A + p_\xi \xi_H + p_Q \log Q_1 + p_0,$$

and the prices of secondary homes at $t = 1$ and $t = 2$ are identical and equal to a log-linear function of ξ_M and $\log P$:

$$\log Q_1 = \log Q_2 = q_\xi \xi_M + q_P \log P + q_0.$$

All coefficients are given in the Internet Appendix. As the immigrants' demand at $t = 2$ for secondary homes is determined by the public information available at $t = 1$, the households can fully anticipate the price of secondary homes Q_2 at $t = 2$ and act as intermediaries to hold secondary homes from $t = 1$ to $t = 2$. Competitive pressure ensures that they earn zero profit and the price of secondary homes Q_1 at $t = 1$ is equal to Q_2.

As a result of the separate supply shocks in the primary and secondary home markets, the prices of primary and secondary homes are not perfectly correlated. The price of primary homes P serves to aggregate the private information of households regarding the strength of the neighborhood A, while the price of secondary homes simply reflects P, together with another supply component $q_\xi \xi_M$, which in turn reveals the supply shock ξ_M. Each household, say household i, treats both prices P and Q_1 as useful signals, in addition to its private signal θ_i, in forming its expectation of A.

Following our discussion of the baseline model, through this informational channel, informational frictions and households' overreaction to the price signals can jointly lead to a humped-shape relationship between the log-price variance of both primary and secondary homes and the supply elasticity. To illustrate this relationship, we again use a numerical example based on the following parameter choices:

$$\tau_\theta = 0.1,\ \tau_A = 1, \phi = 0.1,\ \psi = 0.6, \eta_c = 0.5, \eta_H = 0.9, \alpha = 1, \tau_\varepsilon = 0.1. \tag{16}$$

The top two panels of Figure 7 depicts the log-price variance of both primary and secondary homes against supply elasticity. It shows humped-shapes for both curves in the presence of informational frictions, consistent with that in Figure 6 for the baseline model.

The households' demand for secondary homes is ultimately driven by the immigrants' learning about the neighborhood strength through the housing prices. As a consequence, the learning effects are particularly important on the households' demand for secondary homes. Thus, in this extended model, the households' demand for secondary homes provides an additional dimension to examine learning effects. Specifically, the demand of household i for primary homes is a log-linear function of its private signal θ_i, $\log P$, and $\log Q_1$, while its demand for secondary homes is a log-linear function of $\log P$ and $\log Q_1$:

$$\begin{aligned} \log H_i &= h_\theta \theta_i + h_P \log P + h_Q \log Q_1 + h_0, \\ \log M_i &= \log M = m_P \log P + m_Q \log Q_1 + m_0, \end{aligned}$$

with all coefficients given in the Internet Appendix. As all households agree on the housing demand of immigrants at $t = 2$, they choose an identical demand schedule for secondary homes. We are particularly interested in the fraction of demand for secondary homes relative to the total housing demand $\frac{M_i}{H_i+M_i}$, as this ratio is directly measurable in the data.

The bottom-left panel of Figure 7 depicts the variance of $\frac{M_i}{H_i+M_i}$, which measures the variability of investment-driven housing demand relative to consumption-driven housing demand with respect to supply elasticity in the presence and absence of informational frictions. In the absence of informational frictions, $Var\left[\frac{M_i}{H_i+M_i}\right]$ is monotonically decreasing with supply elasticity. This pattern is intuitive and reflects how the cost effect of a higher housing price impacts primary home purchases more than secondary home purchases. From the Internet Appendix, the loading of primary home demand $\log H$ on the supply shock ζ_H is $\frac{k(1+\psi-\eta_H)}{1+k\eta_H}$, where $\eta_H \in (0, 1)$ is the degree of housing-consumption complementarity, while for secondary home demand $\log M$ it is $\frac{k}{1+k}$. The primary home demand is, therefore, more variable than secondary home demand for a given supply shock, and this difference is increasing with the supply elasticity of the neighborhood. Consequently, in areas with more elastic supply, the fraction of secondary home purchases is less variable.

Interestingly, in the presence of informational frictions, Figure 7 shows a humped-shape pattern of $Var\left[\frac{M_i}{H_i+M_i}\right]$ with respect to supply elasticity. This humped-shape highlights the learning effects on the households' demand for secondary homes. Building on the same insight from our earlier discussion, the demand for secondary homes is most variable in an intermediate range of supply elasticity because the joint effects of informational frictions and the overreaction of the households and immigrants to the price signals are most influential in affecting their expectation of the neighborhood strength, and thus the households' demand for secondary homes.

The non-monotonic relationship between the variability of the fraction of secondary-home demand relative to total demand and housing supply elasticity is in sharp contrast to the monotonic relationship in the perfect-information benchmark. This non-monotonic relationship provides a new prediction for us to explore in the data.

Figure 7 also highlights a second salient feature in the presence of informational frictions. The variance of housing prices and the variance of the fraction of secondary-home demand exhibit similar hump-shaped patterns across supply elasticity, which suggests that we should expect to find a positive correlation between them in the data. The bottom-right panel of Figure 7, by displaying a scatter plot of these two variances across supply elasticities, illustrates this positive correlation, which provides a second new prediction for us to test in the data.

3.2 Empirical evidence

In this subsection, we examine the two predictions provided by the model extension using data from the recent U.S. housing boom: 1) whether during the boom period of 2004-2006 the share of non-owner-occupied home purchases in the total home purchases had the greatest increases relative to the pre-boom period of 2001-2003 in counties with intermediate supply elasticities, and 2) whether counties with greater increases in the share of non-owner-occupied home purchases during the boom also experienced larger price increases in 2004-2006 and larger price decreases during the bust period of 2007-2009.

We construct the share of non-owner-occupied home purchases at the county level from the "Home Mortgage Disclosure Act" (HMDA) data set. The HMDA has comprehensive coverage for mortgage applications and origination in the U.S. We use mortgages originated for home purchases. HMDA data identifies owner occupancy for each individual mortgage. We then aggregate the HMDA data to the county level and calculate the fraction of mortgage origination for non-owner-occupied homes in the total mortgage origination as our measure of the share of secondary home purchases.

Figure 8 depicts the share of non-owner-occupied home purchases for the U.S. and for three cities, New York, Las Vegas, and Charlotte. At a national level, the share of non-owner-occupied home purchases rose steadily from a modest level of 7% in 2000 to peak at a level above 15% in 2005. It then fell gradually to less than 10% in 2010. The peak of the share of non-owner-occupied home purchases in 2005 was slightly in advance of the peak of the national home price index in 2006, as shown in Figure 1. Nevertheless, the rise and fall of the share of non-owner-occupied home purchases were roughly in sync with the boom and bust of home prices.

Across the three cities, it is interesting to note that Las Vegas had the most dramatic rise and fall in the share of non-owner-occupied home purchases, followed by Charlotte, and with New York having the most modest rise and fall. The most variable share of non-owner-occupied home purchases experienced by Las Vegas is particularly interesting as Vegas also had the most dramatic price cycle among these cities.

We now systematically examine the share of non-owner-occupied home purchases across counties with different housing supply elasticities. We focus on the change in the fraction of non-owner-occupied home purchases from the pre-boom period in 2001-2003 to the boom period in 2004-2006.

In Figure 9, we sort the counties in our sample into three groups in the top panel and ten groups in the bottom panel using the Saiz's elasticity measure, with each group having the same number of counties. The top panel shows that the change in the fraction of non-owner-occupied home purchases is almost the same between the inelastic and the middle groups, and smaller in the elastic group. As we discussed before, this coarse grouping might hide finer non-monotonicity. Indeed, the bottom panel shows that the change in the fraction of non-owner-occupied home purchases displays a non-monotonic pattern across ten elasticity groups with the largest share of non-owner-occupied home purchases in groups 3 and 4. This non-monotonic pattern is consistent with the first prediction of the extended model.

In Figure 10, we sort the counties into elasticity groups each with an equal population rather than number of counties. Across either the three groups shown in the top panel or the ten groups shown in the bottom panel, there is a non-monotonic pattern in the change of the share of non-owner-occupied home purchases across the elasticity groups, with the change peaking in the middle of the groups.

To further examine whether the largest change in the fraction of non-owner-occupied home purchases in the middle elasticity groups is robust to controlling for other fundamental factors, we also adopt a regression approach in Table 2. Similar to the regressions reported in Table 1, we regress the change in the share of non-owner-occupied home purchases from 2001-2003 to 2004-2006 on two dummy variables that indicate whether a county is in the middle elastic group or the elastic group, which are constructed in the top panel of Figure 10, together with a list of control variables. This regression implicitly uses the inelastic group as the benchmark for the middle and elastic groups. The control variables include the fraction of subprime households in the county in 2005, the population change, and annualized per capita income change. Columns 1 and 2 of Table 2 report the regressions without and with the controls. In either regression specification, we observe the middle group has a significantly larger change in the share of non-owner-occupied home purchases than the other groups. Furthermore, none of the control variables is significant except the annualized per capita income change in 2004-2006.

Taken together, Figures 9-10 and Table 2 confirm the first prediction of the extended model that there is a non-monotonic relationship between the variability of the share of non-owner-occupied home purchases and housing supply elasticity.

Figures 11 and Table 3 provide evidence of the second prediction. The regressions in Table 3 show that change in the share of non-owner occupied home purchases from 2001-2003 to 2004-2006 is positively correlated with the size of the housing price boom in 2004-2006, and negatively correlated with the size of the housing price bust in 2007-2009. These results are robust to the inclusion of the control variables that are included in the test of the first prediction. The two panels of Figure 11 graphically illustrate these correlations and show that these comovements are a broad feature of the data rather than driven by a few outlying counties. Our empirical analysis thus confirms the second prediction of the extended model that there is a positive correlation between the variability of housing prices and the variability of the share of non-owner occupied home purchases during the recent U.S. housing cycle.

4 Conclusion

This paper highlights a non-monotonic relationship between the magnitude of housing cycles and housing supply elasticity in the cross-section of U.S. county data during the U.S. housing cycle of the 2000's. We develop a tractable model to analyze information aggregation and learning in housing markets to explain this phenomenon. In the presence of pervasive informational frictions regarding economic strength and housing supply of a neighborhood, households face a realistic problem in learning about these fundamental variables with housing prices serving as important signals. Our model highlights how the households' learning interacts with characteristics endemic to local housing supply and demand to impact housing price dynamics. In particular, supply elasticity affects not only housing supply but also the informational noise in the price signal for the households' learning of the neighborhood strength. Our model predicts that housing price and share of investment home purchases are both most variable in areas with intermediate supply elasticities, and that these variances are positively correlated, which is supported by our empirical analysis.

Appendix Proofs of Propositions

A.1 Proof of Proposition 1

The first order conditions for household i's choices of H_i and l_i at an interior point are

$$H_i \;:\; \frac{(1-\eta_c)(1-\eta_H)}{H_i} E\left[U\left(\{H_j\}_{j\in[0,1]}, C_i\right)\Big| \mathcal{I}_i\right] = PE\left[\frac{\eta_H}{C_i} U\left(\{H_j\}_{j\in[0,1]}, C_i\right)\Big| \mathcal{I}_i\right] \quad (17)$$

$$l_i \;:\; l_i^{\psi} = E\left[\frac{\eta_H}{C_i} U\left(\{H_j\}_{j\in[0,1]}, C_i\right) e^{A_i}\Big| \mathcal{I}_i\right]. \quad (18)$$

Imposing $C_i = e^{A_i} l_i$ in equation (17), one arrives at

$$PH_i = \frac{(1-\eta_c)(1-\eta_H)}{\eta_H} \frac{E\left[\left(\int_{[0,1]/i} H_j dj\right)^{\eta_c(1-\eta_H)} e^{\eta_H A_i}\Big| \mathcal{I}_i\right]}{E\left[\left(\int_{[0,1]/i} H_j dj\right)^{\eta_c(1-\eta_H)} e^{(\eta_H-1) A_i}\Big| \mathcal{I}_i\right]} l_i.$$

From equation (18), it follows that

$$l_i = \left\{\eta_H E\left[\left\{\frac{1}{1-\eta_H}\left(\frac{H_i}{1-\eta_c}\right)^{1-\eta_c}\left(\frac{\int_{[0,1]/i} H_j dj}{\eta_c}\right)^{\eta_c}\right\}^{1-\eta_H}\left(\frac{1}{\eta_H} e^{A_i}\right)^{\eta_H}\Bigg| \mathcal{I}_i\right]\right\}^{\frac{1}{1+\psi-\eta_H}},$$

from which we see that

$$\begin{aligned}
\log H_i \;=\; & \frac{2+\psi-\eta_H}{\psi+(1-\eta_H)\eta_c} \log E\left[\left(\int_{[0,1]/i} H_j dj\right)^{\eta_c(1-\eta_H)} e^{\eta_H A_i}\Big| \mathcal{I}_i\right] - \frac{1+\psi-\eta_H}{\psi+(1-\eta_H)\eta_c} \log P \\
& - \frac{1+\psi-\eta_H}{\psi+(1-\eta_H)\eta_c} \log E\left[\left(\int_{[0,1]/i} H_j dj\right)^{\eta_c(1-\eta_H)} e^{(\eta_H-1) A_i}\Big| \mathcal{I}_i\right] \\
& + \frac{\psi}{\psi+(1-\eta_H)\eta_c} \log\left((1-\eta_c)\left(\frac{1-\eta_H}{\eta_H}\right)\left(\frac{1-\eta_c}{\eta_c}\right)^{\frac{\eta_c(1-\eta_H)}{\psi}}\right).
\end{aligned}$$

Note that integrating over the continuum of other households' housing choices does not change when sets of measure zero are substracted from it. We then obtain equation (5).

A.2 Proof of Proposition 2

We first conjecture that each household's housing purchasing and the housing price take the following log-linear forms:

$$\log H_i = h_P \log P + h_\theta \theta_i + h_0, \tag{19}$$

$$\log P = p_A A + p_\xi \xi + p_0, \tag{20}$$

where the coefficients h_0, h_P, h_θ, p_0, p_A, and p_ξ will be determined by equilibrium conditions.

Given the conjectured functional form for H_i, we can expand equation (5). It follows that

$$E\left[\left(\int_{[0,1]} H_j dj\right)^{\eta_c(1-\eta_H)} e^{\eta_H A_i}\middle| \mathcal{I}_i\right]$$
$$= e^{\eta_c(1-\eta_H)\left(h_0+h_P\log P+\frac{1}{2}h_\theta^2\tau_\theta^{-1}\right)+\frac{1}{2}\eta_H^2\tau_\varepsilon^{-1}} E\left[e^{(\eta_H+\eta_c(1-\eta_H)h_\theta)A}\middle|\mathcal{I}_i\right],$$

where we use the fact that A is independent of ε_j and exploit the Law of Large Number for the continuum when integrating over households, which still holds if we subtract sets of measure 0 from the integral. A similar expression obtains for $E\left[\left(\int_{[0,1]} H_j dj\right)^{\eta_c(1-\eta_H)} e^{(\eta_H-1)A_i}\middle| \mathcal{I}_i\right]$

Define

$$q \equiv \frac{\log P - p_0 - p_\xi \bar{\xi}}{p_A} = A + \frac{p_\xi}{p_A}\left(\xi - \bar{\xi}\right),$$

which is a sufficient statistic of information contained in P. Then, conditional on observing its own signal θ_i and the housing price P, household i's expectation of A is

$$E\left[A \mid \theta_i, \log P\right] = E\left[A \mid \theta_i, q\right] = \frac{1}{\tau_A + \tau_\theta + \frac{p_A^2}{p_\xi^2}\frac{1}{(\phi k)^2}}\left(\tau_A \bar{A} + \tau_\theta \theta_i + \frac{p_A^2}{p_\xi^2}\frac{1}{(\phi k)^2} q\right),$$

and its conditional variance of A is

$$Var\left[A \mid \theta_i, \log P\right] = \left(\tau_A + \tau_\theta + \frac{p_A^2}{p_\xi^2}\frac{1}{(\phi k)^2}\right)^{-1}.$$

Therefore,

$$\log E\left[e^{(\eta_H+\eta_c(1-\eta_H)h_\theta)A}\middle|\mathcal{I}_i\right]$$
$$= (\eta_H + \eta_c(1-\eta_H)h_\theta)\left(\tau_A + \tau_\theta + \frac{p_A^2}{p_\xi^2}\frac{1}{(\phi k)^2}\right)^{-1}\left(\tau_A \bar{A} + \tau_\theta \theta_i + \frac{p_A^2}{p_\xi^2}\frac{1}{(\phi k)^2} q\right)$$
$$+\frac{1}{2}(\eta_H + \eta_c(1-\eta_H)h_\theta)^2\left(\tau_A + \tau_\theta + \frac{p_A^2}{p_\xi^2}\frac{1}{(\phi k)^2}\right)^{-1}.$$

Then,

$$
\begin{aligned}
& \log E\left[\left.\left(\int_{[0,1]} H_j dj\right)^{\eta_c(1-\eta_H)} e^{\eta_H A_i}\right| \mathcal{I}_i\right] \\
= & \left(\eta_H+\eta_c\left(1-\eta_H\right) h_\theta\right)\left(\tau_A+\tau_\theta+\frac{p_A^2}{p_\xi^2} \frac{1}{(\phi k)^2}\right)^{-1} \\
& \cdot\left(\tau_A \bar{A}+\tau_\theta \theta_i+\frac{p_A}{p_\xi^2} \frac{1}{(\phi k)^2}\left(\log P-p_0-p_\xi \bar{\xi}\right)\right) \\
& +\eta_c\left(1-\eta_H\right)\left(h_0+h_P \log P+\frac{1}{2} h_\theta^2 \tau_\theta^{-1}\right)+\frac{1}{2} \eta_H^2 \tau_\varepsilon^{-1} \\
& +\frac{1}{2}\left(\eta_H+\eta_c\left(1-\eta_H\right) h_\theta\right)^2\left(\tau_A+\tau_\theta+\frac{p_A^2}{p_\xi^2} \frac{1}{(\phi k)^2}\right)^{-1} .
\end{aligned}
$$

Substituting this expression into equation (5) and matching coefficients with the conjectured log-linear form in (19), it follows that

$$
h_\theta = \frac{1+\psi+\eta_c\left(1-\eta_H\right) h_\theta}{\psi+\left(1-\eta_H\right) \eta_c}\left(\tau_A+\tau_\theta+\frac{p_A^2}{p_\xi^2} \frac{1}{(\phi k)^2}\right)^{-1} \tau_\theta, \tag{21}
$$

$$
h_P = \frac{1+\psi+\eta_c\left(1-\eta_H\right) h_\theta}{\psi}\left(\tau_A+\tau_\theta+\frac{p_A^2}{p_\xi^2} \frac{1}{(\phi k)^2}\right)^{-1} \frac{p_A}{p_\xi^2} \frac{1}{(\phi k)^2}-\frac{1+\psi-\eta_H}{\psi}, \tag{22}
$$

$$
\begin{aligned}
h_0 = & \frac{1+\psi+\eta_c\left(1-\eta_H\right) h_\theta}{\psi+\left(1-\eta_H\right) \eta_c}\left(\tau_A+\tau_\theta+\frac{p_A^2}{p_\xi^2} \frac{1}{(\phi k)^2}\right)^{-1}\left(\tau_A \bar{A}-\frac{p_A}{p_\xi^2} \frac{1}{(\phi k)^2}\left(p_0+p_\xi \bar{\xi}\right)\right) \\
& +\frac{\eta_c\left(1-\eta_H\right)}{\psi+\left(1-\eta_H\right) \eta_c}\left(h_0+\frac{1}{2} h_\theta^2 \tau_\theta^{-1}\right)+\frac{1}{2} \frac{(1+\psi) \eta_H-\left(1+\psi-\eta_H\right)\left(1-\eta_H\right)}{\psi+\left(1-\eta_H\right) \eta_c} \tau_\varepsilon^{-1} \\
& +\frac{1}{2} \frac{1+\psi+\eta_c\left(1-\eta_H\right) h_\theta}{\psi+\left(1-\eta_H\right) \eta_c}\left(\eta_H+\eta_c\left(1-\eta_H\right) h_\theta\right)\left(\tau_A+\tau_\theta+\frac{p_A^2}{p_\xi^2} \frac{1}{(\phi k)^2}\right)^{-1} \\
& +\frac{1}{2} \frac{1+\psi-\eta_H}{\psi+\left(1-\eta_H\right) \eta_c}\left(\eta_H-1+\eta_c\left(1-\eta_H\right) h_\theta\right)\left(\tau_A+\tau_\theta+\frac{p_A^2}{p_\xi^2} \frac{1}{(\phi k)^2}\right)^{-1} \\
& +\frac{\psi}{\psi+\left(1-\eta_H\right) \eta_c} \log \left(\left(1-\eta_c\right)\left(\frac{1-\eta_H}{\eta_H}\right)\left(\frac{1-\eta_c}{\eta_c}\right)^{\frac{\eta_c\left(1-\eta_H\right)}{\psi}}\right) .
\end{aligned} \tag{23}
$$

By aggregating households' housing demand and the builders' supply and imposing market clearing in the housing market, we have

$$
h_0+h_P\left(p_0+p_A A+p_\xi \xi\right)+h_\theta A+\frac{1}{2} h_\theta^2 \tau_\theta^{-1}=\xi+k\left(p_0+p_A A+p_\xi \xi\right) .
$$

Matching coefficients of the two sides of the equation leads to the following three conditions:

$$\begin{aligned} h_0 + h_P p_0 + \frac{1}{2} h_\theta^2 \tau_\theta^{-1} &= k p_0, && (24) \\ h_P p_A + h_\theta &= k p_A, && (25) \\ h_P p_\xi &= 1 + k p_\xi. && (26) \end{aligned}$$

It follows from equation (26) that

$$p_\xi = -\frac{1}{k - h_P}, \tag{27}$$

and further from equation (25) that

$$p_A = \frac{h_\theta}{k - h_P}. \tag{28}$$

Thus, by taking the ratio of equations (28) and (27), we arrive at

$$\frac{p_A}{p_\xi} = -h_\theta.$$

Substituting $\frac{p_A}{p_\xi} = -h_\theta$ into equation (21), and defining $b = -\frac{p_A}{p_\xi}$, we arrive at

$$\frac{1}{(\phi k)^2} b^3 + \left(\tau_A + \frac{\psi}{\psi + (1 - \eta_H)\,\eta_c} \tau_\theta \right) b - \frac{1 + \psi}{\psi + (1 - \eta_H)\,\eta_c} \tau_\theta = 0. \tag{29}$$

We see from equation (29) that b has at most one positive root since the above 3rd order polynomial has only one sign change, by Descartes' Rule of Signs. By setting $b \to -b$, we see that there is no sign change, and therefore b has no negative root. Furthermore, by the Fundamental Theorem of Algebra, the roots of the polynomial (29) exist. Thus, it follows that equation (29) has only one real, nonnegative root $b \geq 0$ and 2 complex roots.[11]

Furthermore, by dropping the cubic term from equation (29), one arrives at an upper bound for b :

$$b \leq \frac{1 + \psi}{\psi} \frac{\tau_\theta}{\left(1 + \frac{(1-\eta_H)\eta_c}{\psi}\right) \tau_A + \tau_\theta}.$$

Since $h_\theta = -\frac{p_A}{p_\xi} = b$, we can recover $h_\theta = b > 0$ and $p_\xi = -\frac{1}{b} p_A < 0$. From equation (22) and $b = -\frac{p_A}{p_\xi}$, it follows that

$$h_P = -\frac{1 + \psi - \eta_H}{\psi} + \frac{1 + \psi + \eta_c\,(1 - \eta_H)\,b}{\psi} \frac{\left(\frac{b}{\phi k}\right)^2}{\tau_A + \tau_\theta + \left(\frac{b}{\phi k}\right)^2} \frac{1}{p_A}. \tag{30}$$

[11]The uniqueness of the positive, real root also follows from the fact that the LHS of the polynomial equation is monotonically increasing in b.

From equation (26), one also has that $h_P = k + p_\xi^{-1}$. Since $p_\xi \leq 0$, it follows that $h_P < k$ whenever $k > 0$.

From $h_\theta = b$ and equations (28) and (30), we arrive at

$$p_A = \frac{\psi}{1+\psi\left(1+k\right)-\eta_H}\left(b + \frac{1+\psi+\eta_c\left(1-\eta_H\right)b}{\psi}\frac{\left(\frac{b}{\phi k}\right)^2}{\tau_A+\tau_\theta+\left(\frac{b}{\phi k}\right)^2}\right) > 0. \tag{31}$$

One arrives at p_ξ from recognizing that $p_\xi = -\frac{1}{b}p_A$. Manipulating equation (29), we recognize that

$$\frac{1+\psi+\eta_c\left(1-\eta_H\right)b}{\psi} = \frac{\psi+\left(1-\eta_H\right)\eta_c}{\psi}\left(\tau_A+\tau_\theta+\left(\frac{b}{\phi k}\right)^2\right)b\tau_\theta^{-1}. \tag{32}$$

Substituting equation (32) into equation (31), and invoking equation (29) to replace $\frac{1}{(\phi k)^2}b^3$, one arrives at

$$p_A = \frac{1+\psi}{1+\psi\left(1+k\right)-\eta_H} - \frac{\psi+\left(1-\eta_H\right)\eta_c}{1+\psi\left(1+k\right)-\eta_H}\tau_\theta^{-1}\tau_A b. \tag{33}$$

and from equation (31), equation (32). and $p_\xi = -\frac{1}{b}p_A$, one also has that

$$p_\xi = -\frac{\psi}{1+\psi\left(1+k\right)-\eta_H} - \frac{\psi+\left(1-\eta_H\right)\eta_c}{1+\psi\left(1+k\right)-\eta_H}\tau_\theta^{-1}\left(\frac{b}{\phi k}\right)^2 < 0.$$

From $h_\theta = b$, $b = -\frac{p_A}{p_\xi}$, and equations (30), (24) and (23), one also finds that

$$\begin{aligned} p_0 \quad = \quad & \frac{1+\psi+\eta_c\left(1-\eta_H\right)b}{1+\psi\left(1+k\right)-\eta_H}\frac{\tau_A\bar{A}+\frac{1}{(\phi k)^2}b\bar{\xi}}{\tau_A+\tau_\theta+\left(\frac{b}{\phi k}\right)^2} + \frac{1}{2}\frac{\psi+\eta_c\left(1-\eta_H\right)}{1+\psi\left(1+k\right)-\eta_H}b^2\tau_\theta^{-1} \\ & +\frac{1}{2}\frac{\left(1+\psi\right)\eta_H-\left(1-\eta_H\right)\left(1+\psi-\eta_H\right)}{1+\psi\left(1+k\right)-\eta_H}\tau_\varepsilon^{-1} \\ & +\frac{1}{2}\frac{1+\psi+\eta_c\left(1-\eta_H\right)b}{1+\psi\left(1+k\right)-\eta_H}\left(\eta_H+\eta_c\left(1-\eta_H\right)b\right)\left(\tau_A+\tau_\theta+\left(\frac{b}{\phi k}\right)^2\right)^{-1} \\ & +\frac{1}{2}\frac{1+\psi-\eta_H}{1+\psi\left(1+k\right)-\eta_H}\left(\eta_H-1+\eta_c\left(1-\eta_H\right)b\right)\left(\tau_A+\tau_\theta+\left(\frac{b}{\phi k}\right)^2\right)^{-1} \\ & +\frac{\psi}{1+\psi\left(1+k\right)-\eta_H}\log\left(\left(1-\eta_c\right)\left(\frac{1-\eta_H}{\eta_H}\right)\left(\frac{1-\eta_c}{\eta_c}\right)^{\frac{\eta_c\left(1-\eta_H\right)}{\psi}}\right). \end{aligned} \tag{34}$$

Given p_0, p_A, and $b = -\frac{p_A}{p_\xi}$, we can recover h_0 from equation (23).

Since we have explicit expressions for all other equilibrium objects as functions of b, and b exists and is unique, it follows that an equilibrium in the economy exists and is unique.

A.3 Proof of Proposition 3

When all households observe A directly, there are no longer information frictions in the economy. Since the households' idiosyncratic productivity components are unobservable, they are now symmetric. Then, it follows that $H_j = H_i = H$. Imposing this symmetry in equation (5), we see that each household's housing demand is then given by

$$\begin{aligned}\log H &= \frac{1+\psi}{\psi}A - \frac{1+\psi-\eta_H}{\psi}\log P + \frac{(1+\psi)\,\eta_H - (1-\eta_H)\,(1+\psi-\eta_H)}{2\psi}\tau_\varepsilon^{-1} \\ &\quad + \log\left((1-\eta_c)\left(\frac{1-\eta_H}{\eta_H}\right)\left(\frac{1-\eta_c}{\eta_c}\right)^{\frac{\eta_c(1-\eta_H)}{\psi}}\right).\end{aligned}$$

By market clearing, $\log H = \xi + k\log P$, it follows that

$$\begin{aligned}\log P &= \frac{1+\psi}{1+\psi\,(1+k)-\eta_H}A - \frac{\psi}{1+\psi\,(1+k)-\eta_H}\xi + \frac{(1+\psi)\,\eta_H - (1-\eta_H)\,(1+\psi-\eta_H)}{2\,(1+\psi\,(1+k)-\eta_H)}\tau_\varepsilon^{-1} \\ &\quad + \frac{\psi}{1+\psi\,(1+k)-\eta_H}\log\left((1-\eta_c)\left(\frac{1-\eta_H}{\eta_H}\right)\left(\frac{1-\eta_c}{\eta_c}\right)^{\frac{\eta_c(1-\eta_H)}{\psi}}\right).\end{aligned}$$

This characterizes the economy in the limit as information frictions dissipate.

To see that the economy with information frictions (finite τ_θ) converges to this perfect-information limit, we consider a sequence of τ_θ that converges to ∞. From equation (29), it follows that, as $\tau_\theta \nearrow \infty$, $b \to \frac{1+\psi}{\psi}$. Since $h_\theta = b$, it follows that

$$h_\theta \to \frac{1+\psi}{\psi}.$$

Taking the limit $\tau_\theta \nearrow \infty$ in equation (31), recognizing that $h_\theta = b$ remains finite in the limit, we see that

$$p_A \to \frac{1+\psi}{1+\psi\,(1+k)-\eta_H}.$$

Since $p_\xi = -\frac{1}{b}p_A$, it follows that

$$p_\xi \to -\frac{\psi}{1+\psi\,(1+k)-\eta_H}.$$

In addition, from equation (30), we find that as $\tau_\theta \nearrow \infty$,

$$h_P \to -\frac{1+\psi-\eta_H}{\psi}$$

Finally, from equations (34) and (23), it follows that

$$
\begin{aligned}
p_0 &\rightarrow \frac{\psi}{1+\psi\left(1+k\right)-\eta_H}\log\left(\left(1-\eta_c\right)\left(\frac{1-\eta_H}{\eta_H}\right)\left(\frac{1-\eta_c}{\eta_c}\right)^{\frac{\eta_c\left(1-\eta_H\right)}{\psi}}\right) \\
&\quad +\frac{1}{2}\frac{\left(1+\psi\right)\eta_H-\left(1-\eta_H\right)\left(1+\psi-\eta_H\right)}{1+\psi\left(1+k\right)-\eta_H}\tau_\varepsilon^{-1}, \\
h_0 &\rightarrow \log\left(\left(1-\eta_c\right)\left(\frac{1-\eta_H}{\eta_H}\right)\left(\frac{1-\eta_c}{\eta_c}\right)^{\frac{\eta_c\left(1-\eta_H\right)}{\psi}}\right)+\frac{\left(1+\psi\right)\eta_H-\left(1-\eta_H\right)\left(1+\psi-\eta_H\right)}{2\psi}\tau_\varepsilon^{-1}.
\end{aligned}
$$

Thus, we see that the economy with information frictions converges to the perfect-information benchmark as $\tau_\theta \nearrow \infty$.

Furthermore, the variance of the log housing price is given by

$$
Var\left[\log P\right]=\left(\frac{\psi k}{1+\psi\left(1+k\right)-\eta_H}\right)^2\left(1+\left(\frac{1+\psi}{\psi k}\right)^2\tau_A^{-1}\right),
$$

from which follows that

$$
\frac{\partial Var\left[\log P\right]}{\partial k}=\frac{2\psi^2 k}{\left(1+\psi\left(1+k\right)-\eta_H\right)^3}\left\{1+\psi-\eta_H-\frac{\left(1+\psi\right)^2}{\psi k}\tau_A^{-1}\right\}.
$$

For $k<\frac{1+\psi}{\psi}\frac{1+\psi}{1+\psi-\eta_H}\tau_A^{-1}$, $\frac{\partial Var[\log P]}{\partial k}<0$. For $k>\frac{1+\psi}{\psi}\frac{1+\psi}{1+\psi-\eta_H}\tau_A^{-1}$, $\frac{\partial Var[\log P]}{\partial k}>0$. Thus it follows that the log housing price is U-shaped in k.

A.4 Proof of Proposition 4

From equation (33), it is clear that

$$
p_A=\frac{1+\psi}{1+\psi\left(1+k\right)-\eta_H}-\frac{\psi+\left(1-\eta_H\right)\eta_c}{1+\psi\left(1+k\right)-\eta_H}\tau_\theta^{-1}\tau_A b<\frac{1+\psi}{1+\psi\left(1+k\right)-\eta_H}.
$$

Thus, it follows that p_A is always lower than its corresponding value in the perfect-information benchmark.

Similarly, since $p_\xi=-\frac{1}{b}p_A$, it follows from equation (31) that we can express p_ξ as

$$
p_\xi=-\frac{\psi}{1+\psi\left(1+k\right)-\eta_H}-\frac{\psi+\left(1-\eta_H\right)\eta_c}{1+\psi\left(1+k\right)-\eta_H}\tau_\theta^{-1}\left(\frac{b}{\phi k}\right)^2<-\frac{\psi}{1+\psi\left(1+k\right)-\eta_H},
$$

which is the corresponding value of p_ξ in the perfect-information benchmark.

A.5 Proof of Proposition 5

Note that b is determined by the polynomial equation (29). We define the LHS of the equation as $G(b)$. By using the Implicit Function Theorem and invoking equation (29), we have

$$\frac{\partial b}{\partial \eta_c} = -\frac{\partial G/\partial \eta_c}{\partial G/\partial b} = -\frac{(1-\eta_H)\,\tau_\theta}{3\frac{1}{(\phi k)^2}b^2 + \tau_A + \frac{\psi}{\psi+(1-\eta_H)\eta_c}\tau_\theta}\frac{1+\psi-\psi b}{(\psi+(1-\eta_H)\,\eta_c)^2}.$$

Since, from Proposition 2, $0 \le b \le \frac{1+\psi}{\psi}\frac{\tau_\theta}{\left(1+\frac{(1-\eta_H)\eta_c}{\psi}\right)\tau_A+\tau_\theta} \le \frac{1+\psi}{\psi}$, it follows that

$$\frac{1+\psi}{\psi} - b \ge 0$$

Thus $\frac{\partial b}{\partial \eta_c} < 0$. Similarly,

$$\frac{\partial b}{\partial \tau_\theta} = -\frac{\partial G/\partial \tau_\theta}{\partial G/\partial b} = \frac{1}{3\frac{1}{(\phi k)^2}b^2 + \tau_A + \frac{\psi}{\psi+(1-\eta_H)\eta_c}\tau_\theta}\frac{1+\psi-\psi b}{\psi+(1-\eta_H)\,\eta_c} > 0.$$

From the expression for p_A in Proposition 2,

$$\frac{\partial p_A}{\partial \eta_c} = -\frac{1-\eta_H}{1+\psi\,(1+k)-\eta_H}\tau_\theta^{-1}\tau_A b - \frac{\psi+(1-\eta_H)\,\eta_c}{1+\psi\,(1+k)-\eta_H}\tau_\theta^{-1}\tau_A\frac{\partial b}{\partial \eta_c}.$$

Then, it follows, subtituting with equation (29), that

$$\frac{\partial p_A}{\partial \eta_c} = -\frac{(1-\eta_H)\,\tau_\theta^{-1}\tau_A b}{1+\psi\,(1+k)-\eta_H}\frac{2\frac{1}{(\phi k)^2}b^3 + \frac{\psi b}{\psi+(1-\eta_H)\eta_c}\tau_\theta}{2\frac{1}{(\phi k)^2}b^3 + \frac{1+\psi}{\psi+(1-\eta_H)\eta_c}\tau_\theta} < 0.$$

Similarly, with respect to τ_θ, we have

$$\begin{aligned}\frac{\partial p_A}{\partial \tau_\theta} &= \frac{\psi+(1-\eta_H)\,\eta_c}{1+\psi\,(1+k)-\eta_H}\tau_\theta^{-2}\tau_A b\left(1-\tau_\theta\frac{1}{b}\frac{\partial b}{\partial \tau_\theta}\right)\\ &= \frac{\psi+(1-\eta_H)\,\eta_c}{1+\psi\,(1+k)-\eta_H}\tau_\theta^{-2}\tau_A b\frac{2\frac{1}{(\phi k)^2}b^3 + \frac{\psi b}{\psi+(1-\eta_H)\eta_c}\tau_\theta}{2\frac{1}{(\phi k)^2}b^3 + \frac{1+\psi}{\psi+(1-\eta_H)\eta_c}\tau_\theta} > 0.\end{aligned}$$

A.6 Proof of Proposition 6

We first consider the limiting case for the economy as $k \to \infty$. Rewrite equation (29) as

$$\left(\frac{b}{\phi k}\right)^3 + \left(\tau_A + \frac{\psi}{\psi+(1-\eta_H)\,\eta_c}\tau_\theta\right)\frac{b}{\phi k} - \frac{1+\psi}{\psi+(1-\eta_H)\,\eta_c}\frac{1}{\phi k}\tau_\theta = 0. \qquad (35)$$

Then it is apparent from equation (35) that, as $k \to \infty$, that either $\frac{b}{\phi k} = 0$ or $\frac{b}{\phi k} = \pm i\sqrt{\tau_A + \frac{\psi}{\psi+(1-\eta_H)\eta_c}\tau_\theta}$. Thus, as $k \to \infty$, one has that $\frac{b}{\phi k} \to 0$, and therefore $\frac{b}{k} \to 0$.

Consequently, from equation (33), $p_A \to 0$ and the housing price is completely driven by the supply shock ξ. From Proposition 2, one has that

$$p_\xi k = -\frac{\psi k}{1+\psi(1+k)-\eta_H} - \frac{\psi+(1-\eta_H)\eta_c}{1+\psi(1+k)-\eta_H}\tau_\theta^{-1}\frac{1}{\phi}\left(\frac{b}{\phi k}\right) b \to -1,$$

since b is bounded from above by $\frac{1+\psi}{\psi}$. Thus, $\log P = -\zeta$.

In addition, from equation (22), then, since $\frac{b}{k} \to 0$ and b is bounded from above by $\frac{1+\psi}{\psi}$, and from below by 0, one has that

$$h_P = -\frac{1+\psi+\eta_c(1-\eta_H)b}{\phi\psi}\left(\tau_A+\tau_\theta+\left(\frac{b}{\phi k}\right)^2\right)^{-1}\frac{b}{\phi k}\frac{1}{p_\xi k} - \frac{1+\psi-\eta_H}{\psi} \to -\frac{1+\psi-\eta_H}{\psi}.$$

From equation (21), it is straightforward to see that, as $k \to \infty$,

$$h_\theta = b \to \frac{1+\psi}{\psi}\frac{\tau_\theta}{\tau_A+\tau_\theta+\frac{(1-\eta_H)\eta_c}{\psi}\tau_A}.$$

Since h_θ remains bounded in the limit, it is easy to see from equation (34) that $p_0 \to 0$ as $k \to \infty$. It further follows from equation (23) that in the limit

$$\begin{aligned} h_0 \;=\; & \frac{1+\psi}{\psi}\left(1+\frac{\eta_c(1-\eta_H)\tau_\theta}{\psi(\tau_A+\tau_\theta)+(1-\eta_H)\eta_c\tau_A}\right)\frac{\tau_A}{\tau_A+\tau_\theta}\bar{A} \\ & +\frac{\eta_c(1-\eta_H)}{2\psi}\left(\frac{\frac{1+\psi}{\psi}}{\tau_A+\tau_\theta+\frac{(1-\eta_H)\eta_c}{\psi}\tau_A}\right)^2\tau_\theta + \frac{1}{2}\frac{(1+\psi)\eta_H-(1+\psi-\eta_H)(1-\eta_H)}{\psi}\tau_\varepsilon^{-1} \\ & +\frac{1+\psi}{2\psi}\left(1+\frac{\eta_c(1-\eta_H)\tau_\theta}{\psi(\tau_A+\tau_\theta)+(1-\eta_H)\eta_c\tau_A}\right)\left(\eta_H+\frac{(1+\psi)\eta_c(1-\eta_H)\tau_\theta}{\psi(\tau_A+\tau_\theta)+(1-\eta_H)\eta_c\tau_A}\right)(\tau_A+\tau_\theta)^{-1} \\ & +\frac{1+\psi-\eta_H}{2\psi}\left(\eta_H-1+\frac{1+\psi}{\psi}\frac{\eta_c(1-\eta_H)\tau_\theta}{\tau_A+\tau_\theta+\frac{(1-\eta_H)\eta_c}{\psi}\tau_A}\right)(\tau_A+\tau_\theta)^{-1} \\ & +\log\left((1-\eta_c)\left(\frac{1-\eta_H}{\eta_H}\right)\left(\frac{1-\eta_c}{\eta_c}\right)^{\frac{\eta_c(1-\eta_H)}{\psi}}\right). \end{aligned} \tag{36}$$

In the case $k \to 0$, it follows from equation (35) that $b \to 0$ and $\frac{b}{k} \to \infty$. From equation (21), it follows that as $k \to 0$ one has that $h_\theta = b \to 0$. Furthermore, from equation (33), one has that

$$p_A \to \frac{1+\psi}{1+\psi-\eta_H}.$$

Since $h_\theta \to 0$, and p_A remain bounded as $k \to 0$, we also see from equation (25), subsituting for the limiting p_A, that $h_P \to 0$. Substituting for p_A in $p_\xi = -\frac{1}{b}p_A$ with equation (31), it follows that

$$p_\xi k = -\frac{\psi k}{1+\psi(1+k)-\eta_H} - \frac{1+\psi+\eta_c(1-\eta_H)b}{1+\psi(1+k)-\eta_H}\frac{\frac{1}{\phi^2}\frac{b}{k}}{\tau_A+\tau_\theta+\left(\frac{b}{\phi k}\right)^2} \to 0,$$

and the demand shock A completely drives the housing price.

Since h_θ remains bounded in the limit, it is easy to see from equation (34) that as $k \to 0$,

$$\begin{aligned} p_0 &= \frac{1}{2}\frac{(1+\psi)\,\eta_H - (1-\eta_H)\,(1+\psi-\eta_H)}{1+\psi-\eta_H}\tau_\varepsilon^{-1} \\ &\quad + \frac{\psi}{1+\psi-\eta_H}\log\left((1-\eta_c)\left(\frac{1-\eta_H}{\eta_H}\right)\left(\frac{1-\eta_c}{\eta_c}\right)^{\frac{\eta_c(1-\eta_H)}{\psi}}\right). \end{aligned} \tag{37}$$

It further follows from equation (24) that in the limit $h_0 \to 0$.

A.7 Proof of Proposition 7

We first prove that p_A is decreasing with ϕ and $p_\xi < 0$ is increasing with ϕ. Note that b is determined by the polynomial equation (29). We define the LHS of the equation as $G(b)$. Comparative statics of b with respect to ϕ reveal, by the Implicit Function Theorem and invoking equation (29), that

$$\begin{aligned} \frac{\partial b}{\partial \phi} &= -\frac{\partial G/\partial \phi}{\partial G/\partial b} = \frac{2\frac{1}{(\phi k)^2}b^3}{3\frac{1}{(\phi k)^2}b^2 + \tau_A + \frac{\psi}{\psi+(1-\eta_H)\eta_c}\tau_\theta}\frac{1}{\phi} \\ &= \frac{2\frac{1}{(\phi k)^2}b^4}{2\frac{1}{(\phi k)^2}b^3 + \frac{1+\psi}{\psi+(1-\eta_H)\eta_c}\tau_\theta}\frac{1}{\phi} > 0. \end{aligned}$$

From the expression for p_A in Proposition 2,

$$\frac{\partial p_A}{\partial \phi} = -\frac{\psi + (1-\eta_H)\,\eta_c}{1+\psi\,(1+k)-\eta_H}\tau_\theta^{-1}\tau_A\frac{\partial b}{\partial \phi} < 0.$$

Furthermore, by the Implicit Function Theorem, it follows that

$$\begin{aligned} \frac{\partial p_\xi}{\partial \phi} &= -\frac{2}{\phi}\frac{\psi + (1-\eta_H)\,\eta_c}{1+\psi\,(1+k)-\eta_H}\tau_\theta^{-1}\left(\frac{b}{\phi k}\right)^2\left(\frac{\phi}{b}\frac{\partial b}{\partial \phi} - 1\right) \\ &= \frac{2}{\phi}\frac{\psi + (1-\eta_H)\,\eta_c}{1+\psi\,(1+k)-\eta_H}\tau_\theta^{-1}\left(\frac{b}{\phi k}\right)^2\frac{\frac{1+\psi}{\psi+(1-\eta_H)\eta_c}\tau_\theta}{2\frac{1}{(\phi k)^2}b^3 + \frac{1+\psi}{\psi+(1-\eta_H)\eta_c}\tau_\theta}. \end{aligned}$$

Since $\phi \in [0,1]$, it follows that $\frac{\partial p_\xi}{\partial \phi} > 0$.

The variance of the housing price $Var[\log P]$ is given by

$$Var[\log P] = p_A^2\tau_A^{-1} + p_\xi^2 k^2,$$

from which follows that

$$\frac{\partial Var[\log P]}{\partial \phi} = 2p_A\tau_A^{-1}\frac{\partial p_A}{\partial \phi} + 2p_\xi k^2\frac{\partial p_\xi}{\partial \phi} < 0,$$

since $p_A \frac{\partial p_A}{\partial \phi} < 0$ and $p_\xi \frac{\partial p_\xi}{\partial \phi} < 0$.

From Proposition 3, the variance of the housing price in the perfect-information benchmark is

$$Var\left[\log P^{perf}\right] = \left(\frac{\psi k}{1+\psi(1+k)-\eta_H}\right)^2 \left(1 + \left(\frac{1+\psi}{\psi k}\right)^2 \tau_A^{-1}\right).$$

It then follows, substituting for p_A and p_ξ with Proposition 2, that

$$\begin{aligned} & Var\left[\log P\right] - Var\left[\log P^{perf}\right] \\ = & \left(p_A^2 - \left(\frac{1+\psi}{1+\psi(1+k)-\eta_H}\right)^2\right)\tau_A^{-1} + \left(p_\xi^2 - \left(\frac{\psi}{1+\psi(1+k)-\eta_H}\right)^2\right)k^2 \\ = & \left(\frac{\psi+(1-\eta_H)\eta_c}{1+\psi(1+k)-\eta_H}\right)^2 \tau_\theta^{-1} b \left(\tau_\theta^{-1}\tau_A b - \frac{2(1+\psi)}{\psi+(1-\eta_H)\eta_c}\right) \\ & + \left(\frac{\psi+(1-\eta_H)\eta_c}{1+\psi(1+k)-\eta_H}\right)^2 \tau_\theta^{-1}\left(\frac{b}{\phi}\right)^2 \left(2\frac{\psi}{\psi+(1-\eta_H)\eta_c} + \tau_\theta^{-1}\left(\frac{b}{\phi k}\right)^2\right), \end{aligned}$$

from which follows, substituting with equation (29), that $Var\left[\log P\right] - Var\left[\log P^{perf}\right] \geq 0$ whenever

$$b \geq \left(\left(\phi^2-1\right)\tau_\theta^{-1}\tau_A + \frac{\psi}{\psi+(1-\eta_H)\eta_c}\right)^{-1}\left(2\phi^2-1\right)\frac{1+\psi}{\psi+(1-\eta_H)\eta_c}. \tag{38}$$

Since $b \geq 0$, it is thus sufficient for $1 - \frac{\psi}{\psi+(1-\eta_H)\eta_c}\frac{\tau_\theta}{\tau_A} \leq \phi^2 \leq \frac{1}{2}$ for the condition in (38) to be satisfied.

When $\phi = 1$, then the condition in (38) becomes $b \geq \frac{1+\psi}{\psi}$. Since

$$0 \leq b \leq \frac{1+\psi}{\psi}\frac{\tau_\theta}{\left(1+\frac{(1-\eta_H)\eta_c}{\psi}\right)\tau_A + \tau_\theta} \leq \frac{1+\psi}{\psi}$$

from Proposition 2, this condition can be satisfied only when $b = \frac{1+\psi}{\psi}$, which is the value of b in the perfect-information benchmark, in which case $Var\left[\log P\right] = Var\left[\log P^{perf}\right]$. Thus, when $\phi = 1$, variance with informational frictions is always less than that of the perfect-information benchmark.

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Figure 1: Case-Shiller Home Price Index

This figure plots the Case-Shiller home price index for the U.S. and three cities, New York, Las Vegas and Charlotte. The price index is deflated by the CPI and normalized to 100 in 2000.

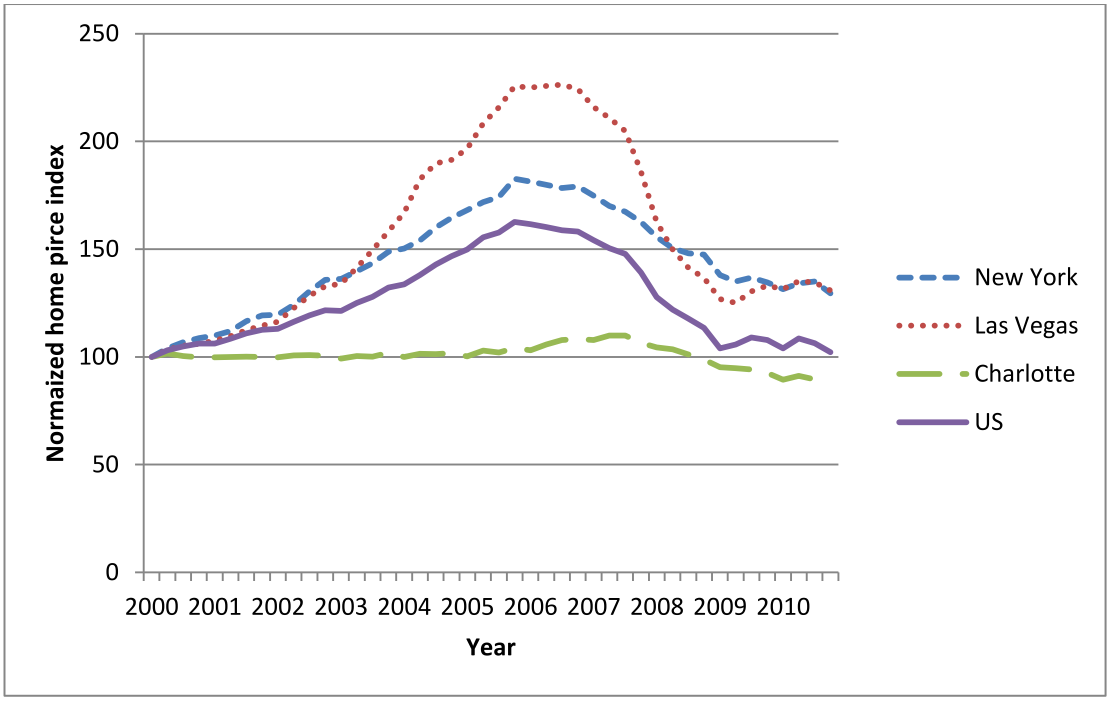

Figure 2: Housing Cycle across Three Elasticity Groups with an Equal Number of Counties

This figure is constructed from sorting the counties in the U.S. into three groups based on the Saiz's housing supply elasticity measure, with each group holding an equal number of counties. The top panel depicts the average housing price expansion during the boom period of 2004-2006 and the average housing price contraction during the bust period of 2007-2009 in each of the groups. The bottom panel depicts the population in each group in the 2000 U.S. census.

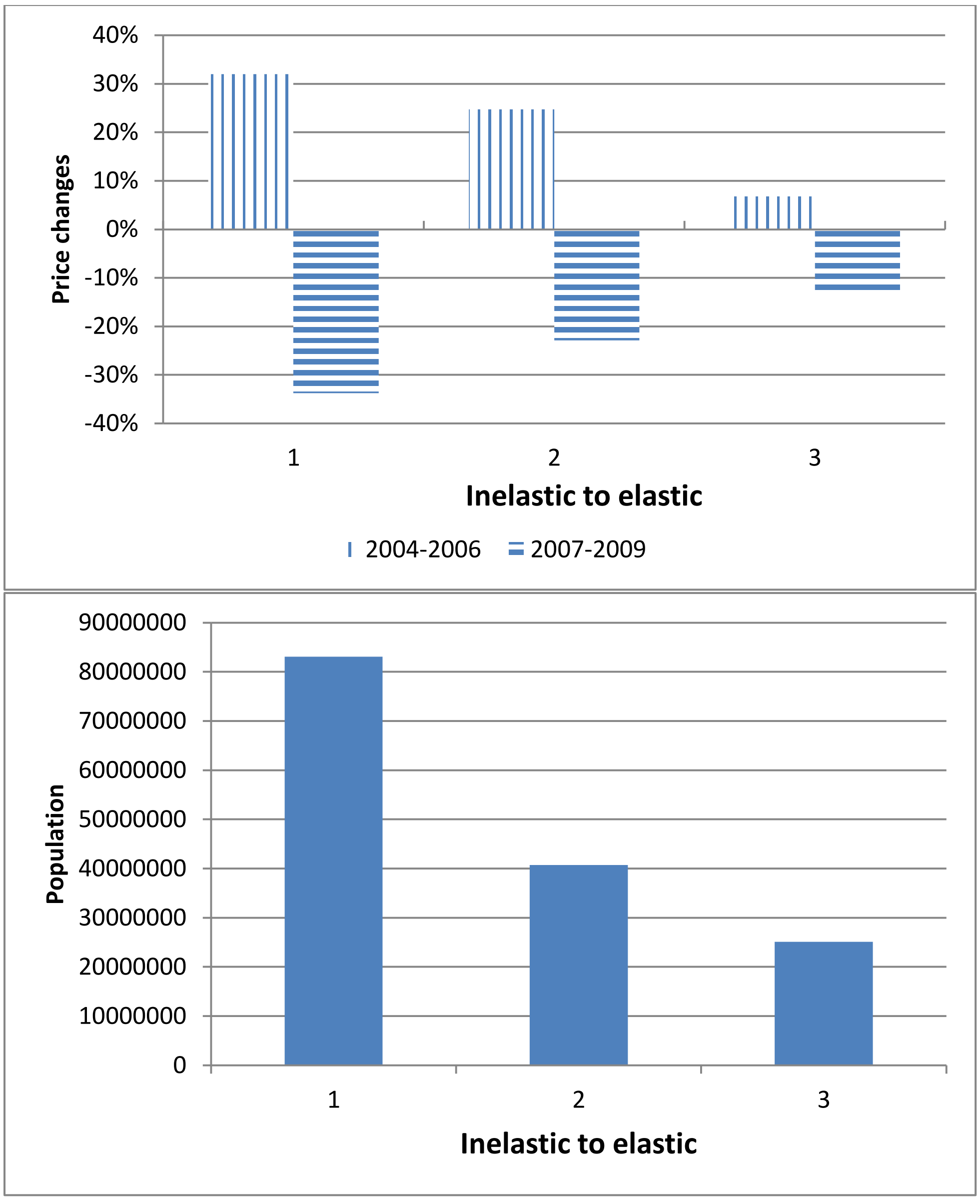

Figure 3: Housing Cycle across Ten Elasticity Groups with an Equal Number of Counties

This figure is constructed from sorting the counties in the U.S. into ten groups based on the Saiz's housing supply elasticity measure, with each group holding an equal number of counties. The top panel depicts the average housing price expansion during the boom period of 2004-2006 and the average housing price contraction during the bust period of 2007-2009 in each of the groups. The bottom panel depicts the population in each group in the 2000 U.S. census.

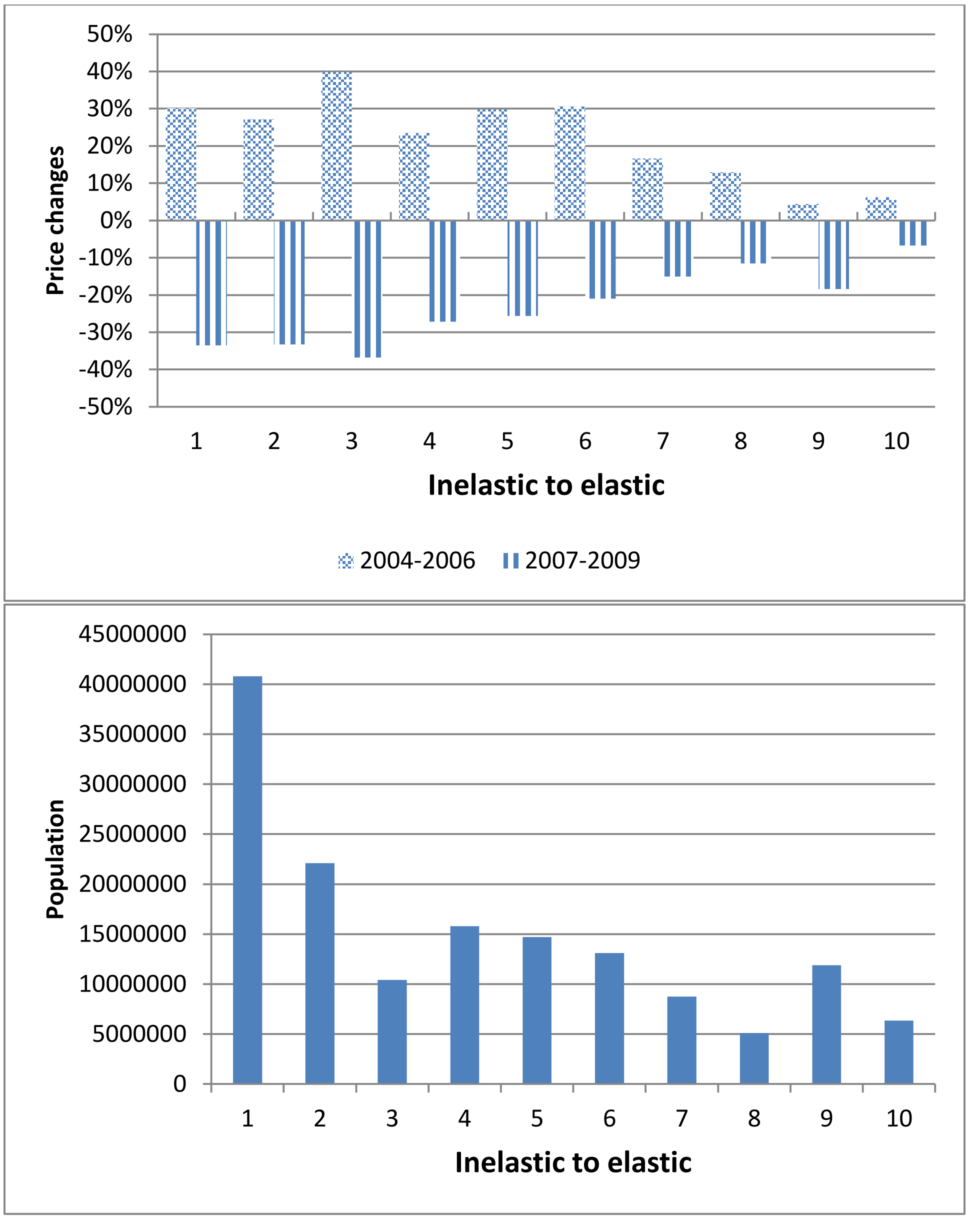

Figure 4: Housing Cycle across Three Elasticity Groups with an Equal Population

This figure is constructed from sorting the counties in the U.S. into three groups based on the Saiz's housing supply elasticity measure, with each group holding an equal population. The top panel depicts the average housing price expansion during the boom period of 2004-2006 and the average housing price contraction during the bust period of 2007-2009 in each of the groups. The bottom panel depicts the population in each group in the 2000 U.S. census.

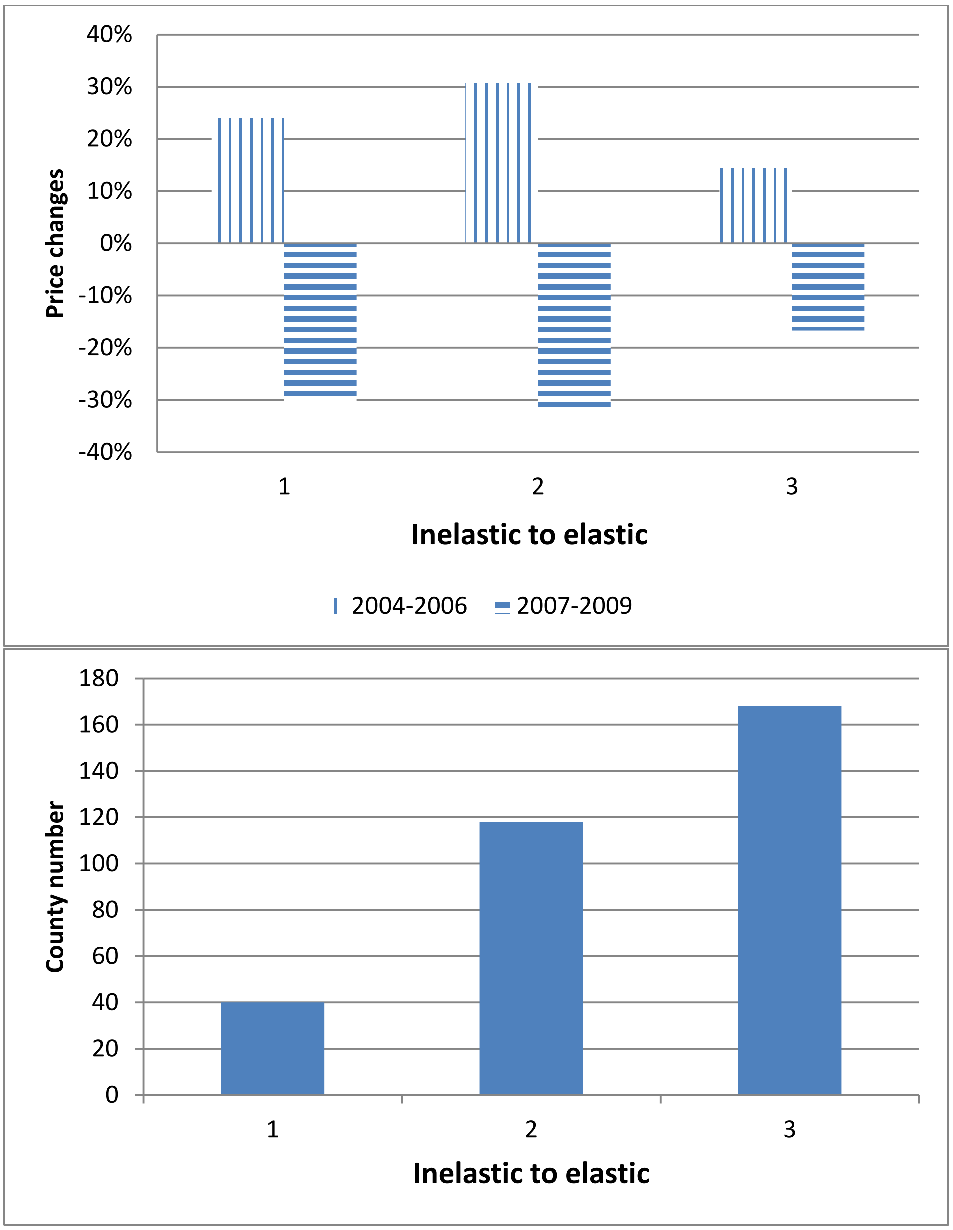

Figure 5: Housing Cycle across Ten Elasticity Groups with an Equal Population

This figure is constructed from sorting the counties in the U.S. into ten groups based on the Saiz's housing supply elasticity measure, with each group holding an equal population. The top panel depicts the average housing price expansion during the boom period of 2004-2006 and the average housing price contraction during the bust period of 2007-2009 in each of the groups. The bottom panel depicts the population in each group in the 2000 U.S. census.

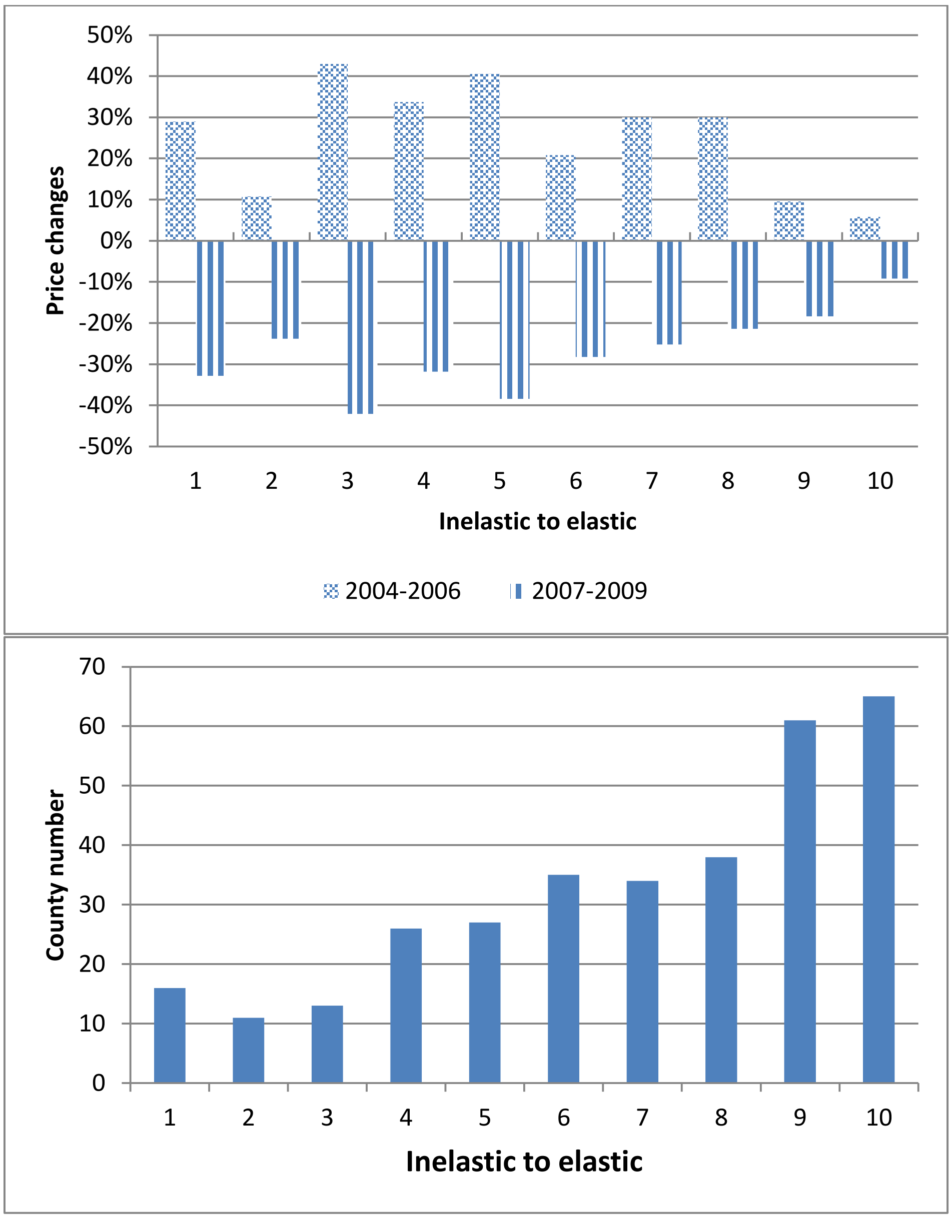

Figure 6: Housing Price Variance in the Baseline Model

This figure depicts the log-price variance in the baseline model against the supply elasticity, based on the following parameters: $\tau_\theta = 0.1$, $\tau_A = 1, \phi = 0.1, \eta_c = 0.5, \psi = 0.6, \eta_H = 0.9$. The solid line depicts the log-price variance in the presence of informational frictions, while the dashed line depicts the log-price variance in the perfect-information benchmark.

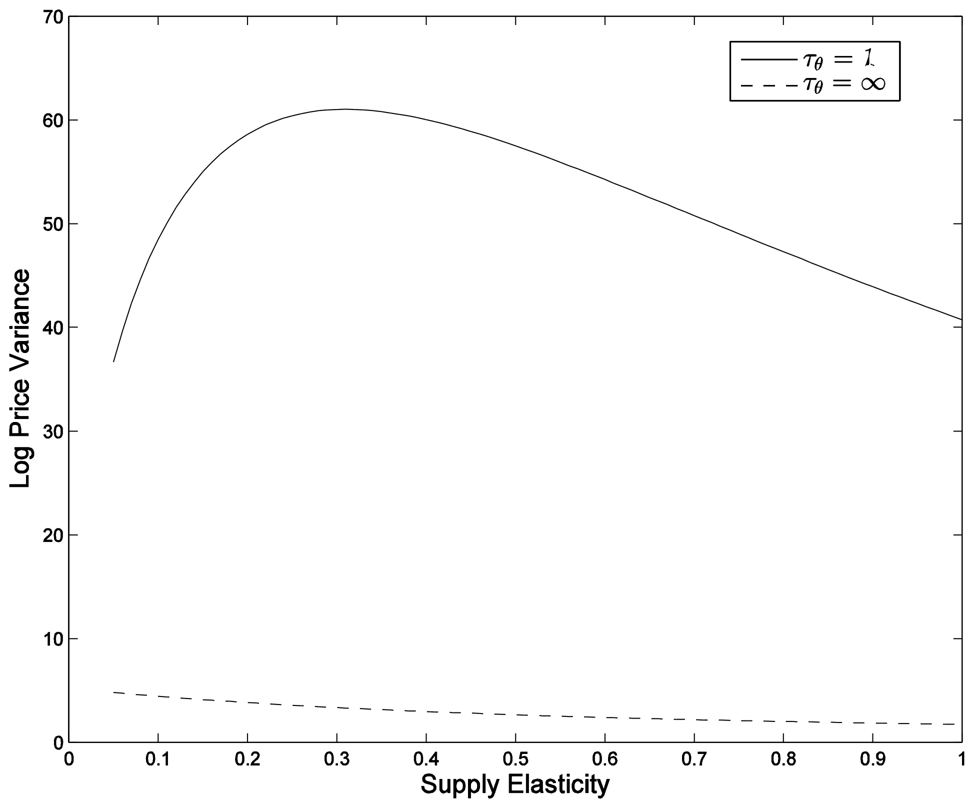

Figure 7: Variance of Primary and Secondary Housing Prices and Fraction of Secondary Homes in the Extended Model

This figure depicts the log-price variance of both primary and secondary homes in the extended model in the top two panels, the variance of the fraction of secondary home demand in the bottom left panel, and a scatter plot of the variance of the fraction and the variance of the secondary housing price in the bottom right panel based on the following parameters: $\tau_\theta = 0.1, \tau_A = 1, \phi = 0.1, \eta_c = 0.5, \psi = 0.6, \eta_H = 0.9, \alpha = 1, \tau_\varepsilon = 0.1$.

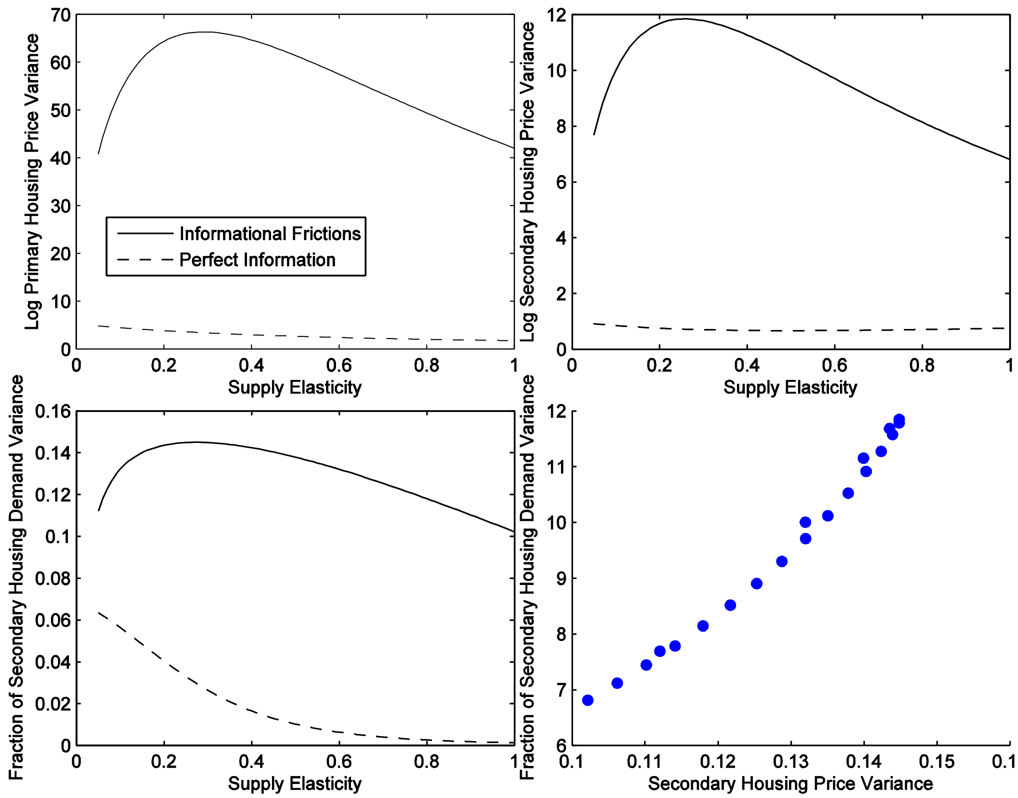

Figure 8: The Share of Non-Owner-Occupied Home Purchases

This figure plots the share of non-owner-occupied home purchases for the U.S. and three cities, New York, Las Vegas and Charlotte.

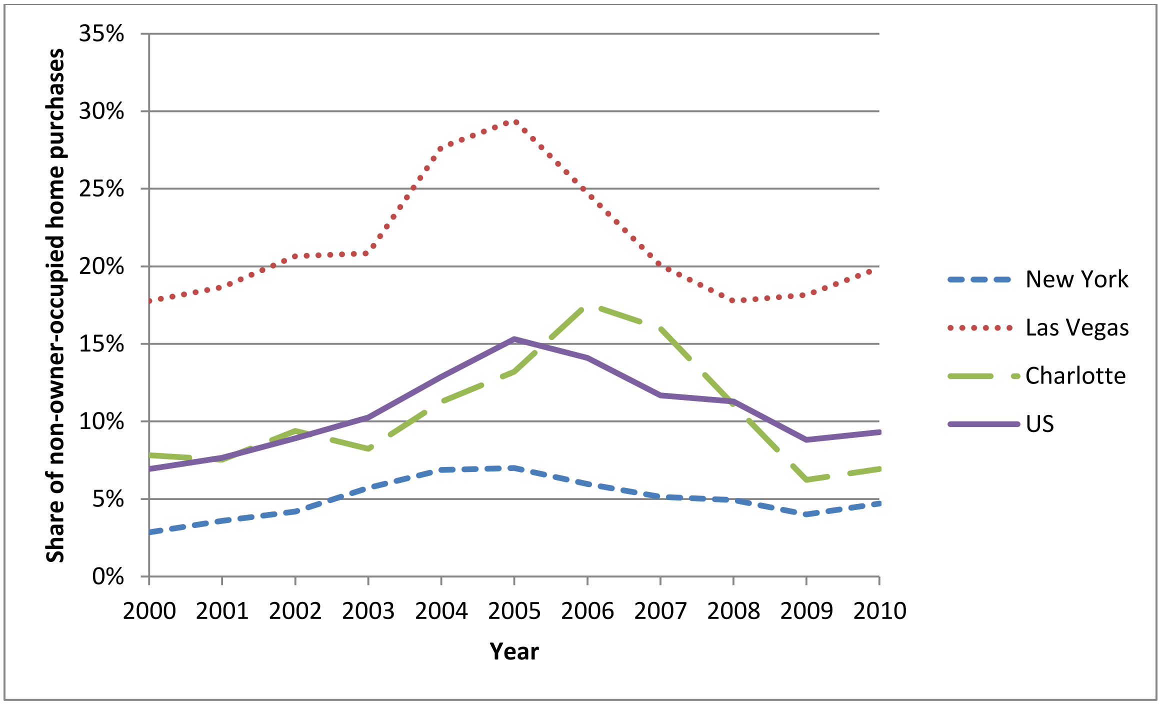

Figure 9: Change in the Fraction of Non-Owner-Occupied Home Purchases from 2001-2003 to 2004-2006 across Elasticity Groups with an Equal Number of Counties

We use the Saiz's (2010) measure of supply elasticity measure to sort the counties in our sample into three groups in the top panel and ten groups in the bottom panel, with each group holding the same number of counties. Each bar measures the change of fraction of non-owner occupied home purchases from 2001-2003 to 2004-2006 in each group. The fraction of non-owner-occupied home purchases in each county is computed from the "Home Mortgage Disclosure Act" data set.

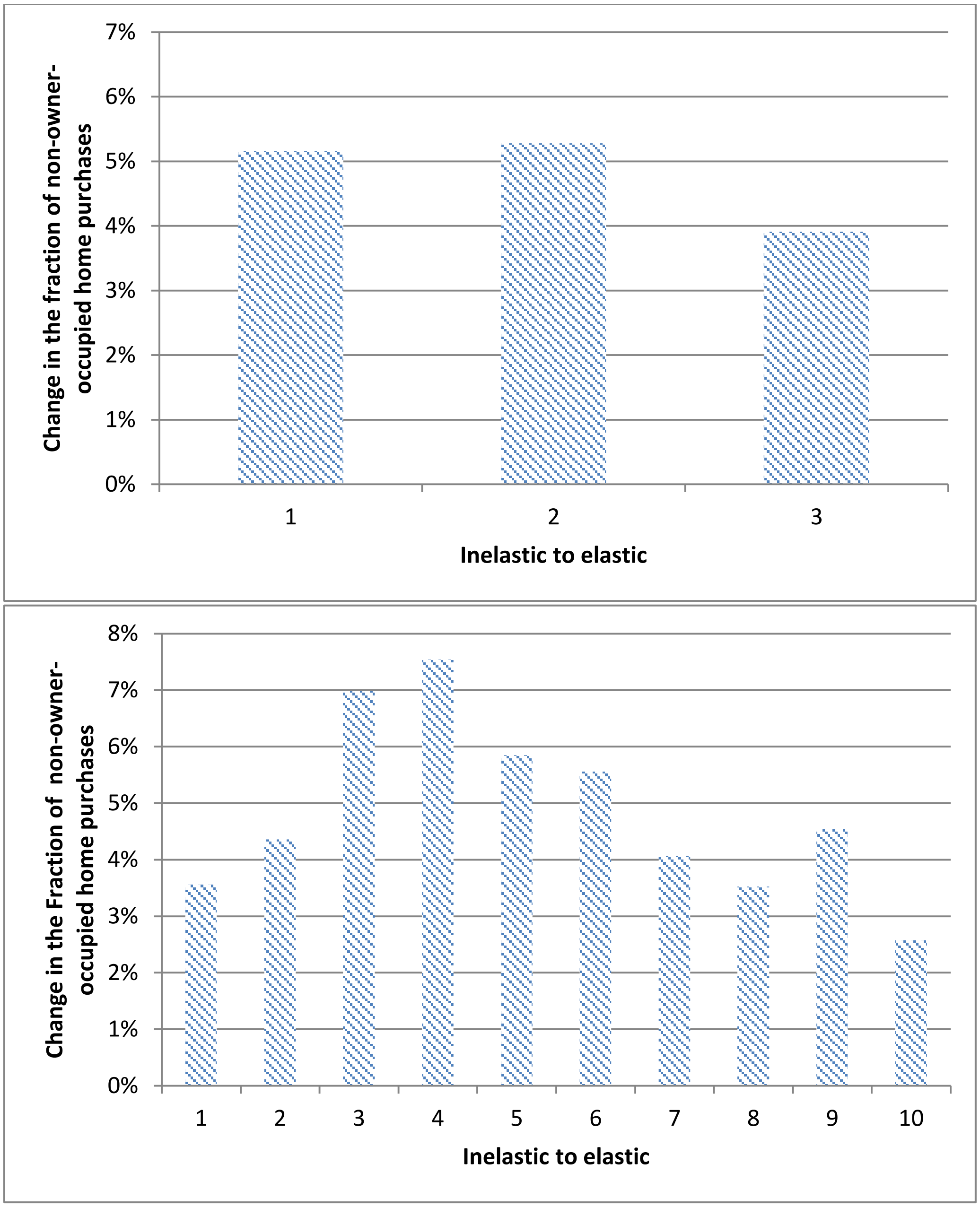

Figure 10: Change in the Fraction of Non-Owner-Occupied Home Purchases from 2001-2003 to 2004-2006 across Elasticity Groups with an Equal Population

We use the Saiz's (2010) measure of supply elasticity measure to sort the counties in our sample into three groups in the top panel and ten groups in the bottom panel, with each group holding the same population. Each bar measures the change of fraction of non-owner occupied home purchases from 2001-2003 to 2004-2006 in each group. The fraction of non-owner-occupied home purchases in each county is computed from the "Home Mortgage Disclosure Act" data set.

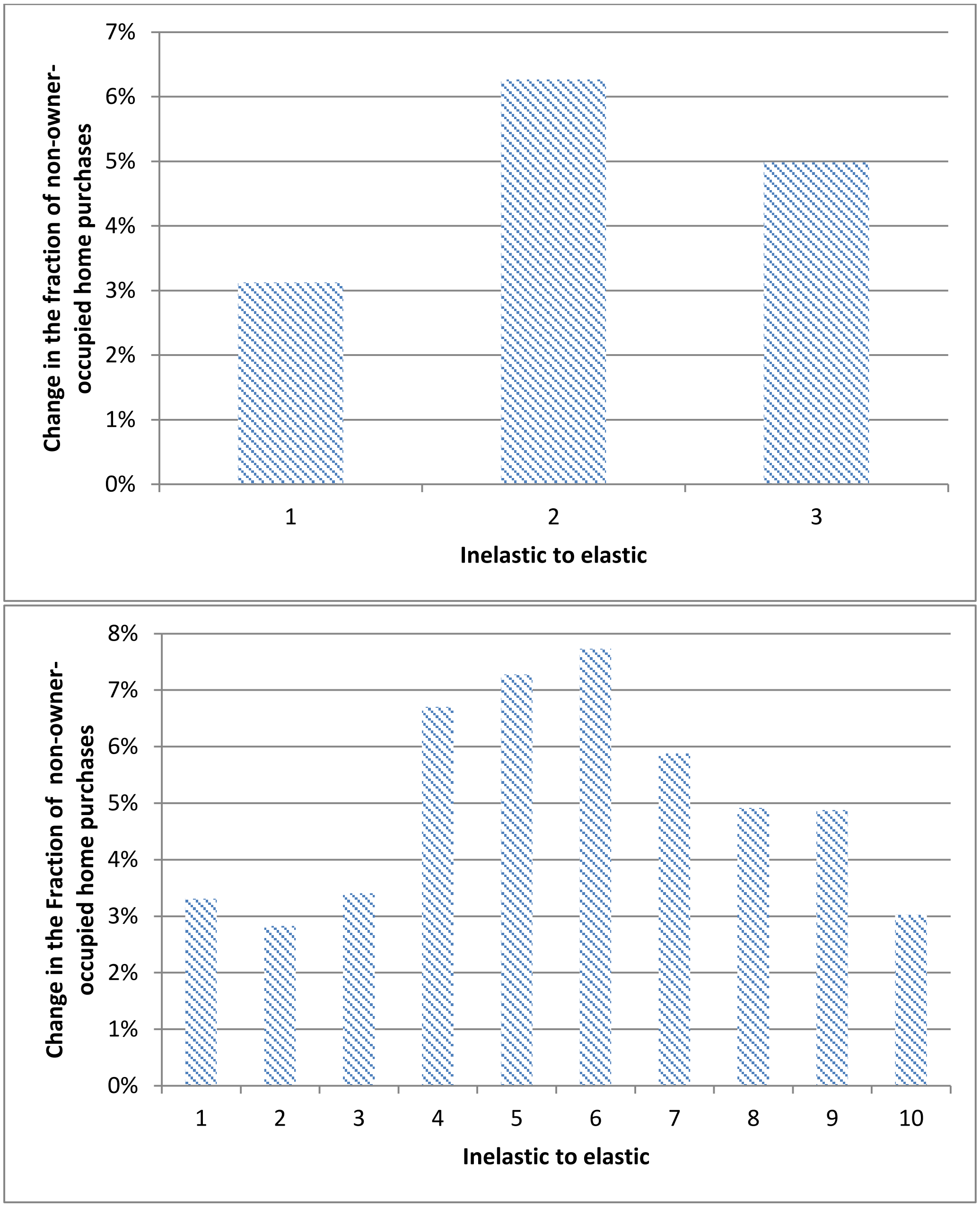

Figure 11: Change in the Fraction of Non-Owner-Occupied Home Purchases from 2001-2003 to 2004-2006 and the Recent Housing Cycle

The top panel plots the average housing price expansion during the boom period of 2004-2006 against the change of fraction of non-owner occupied home purchases from 2001-2003 to 2004-2006; the bottom panel the average housing price contraction during the bust period of 2007-2009 against the change of fraction of non-owner occupied home from 2001-2003 to 2004-2006.

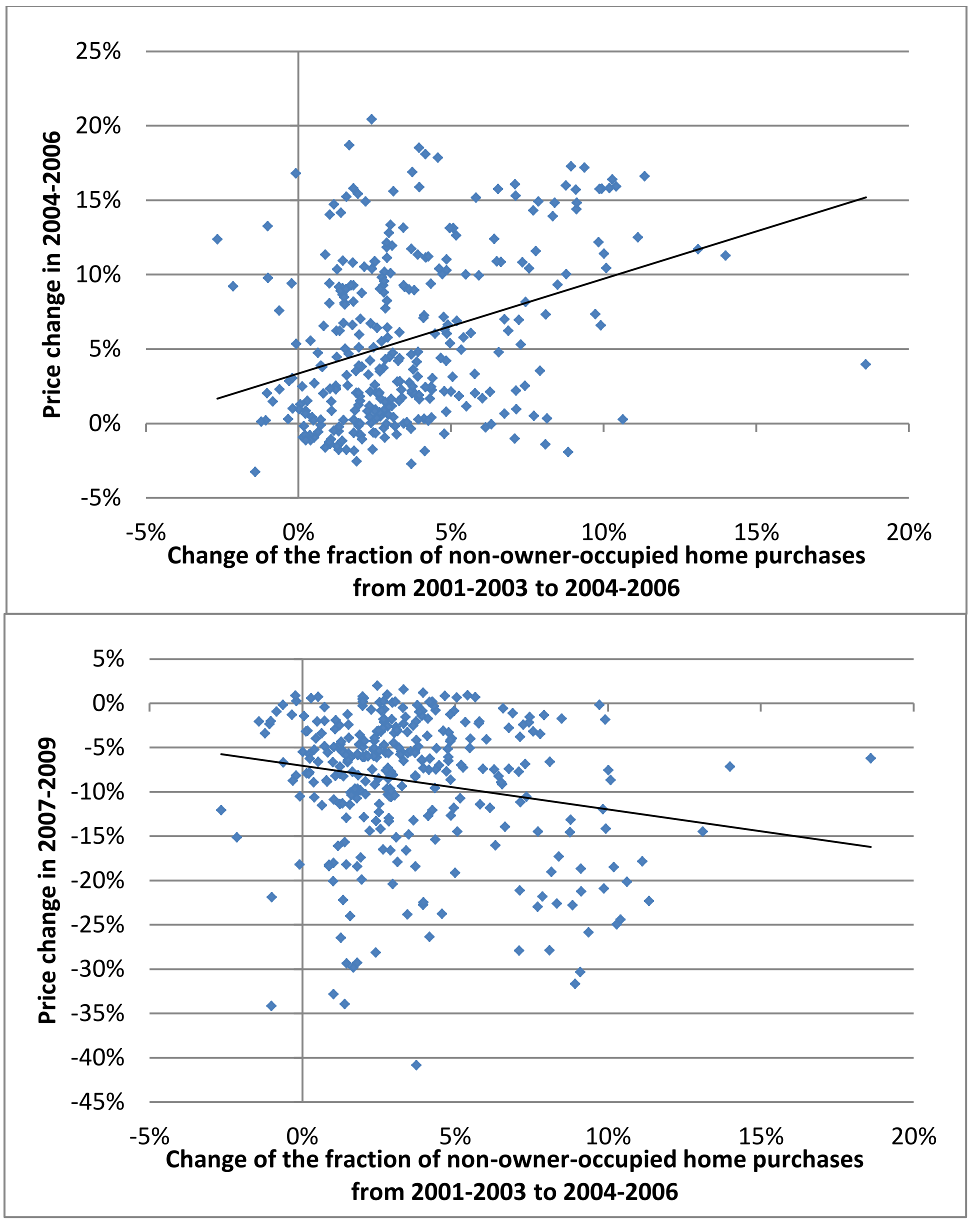

Table 1: Housing Boom and Bust during the Recent Cycle

This table presents coefficient estimates from regressing the change in real house price during 2004-2006 (housing boom period) and during 2007-2009 (housing bust period) on the dummies indicating whether a county is in the middle-elasticity group or the elastic group, with the inelastic group as the benchmark and a list of control variables. Robust standard errors are in parentheses. ***, **, * indicate coefficient estimates statistically distinct from 0 at the 1%, 5%, and 10% levels, respectively.

	(1)	(2)	(3)	(4)
	Annualized real house price change in 2004-2006		Annualized real house price change in 2007-2009	
Middle group dummy	0.0145*	0.0233***	-0.00559	-0.0387***
	(0.00871)	(0.00857)	(0.0119)	(0.0107)
Elastic group dummy	-0.0308***	-0.0173**	0.0591***	0.0125
	(0.00798)	(0.00834)	(0.0105)	(0.0102)
Fraction of subprime households in 2005		0.173***		-0.605***
		(0.0475)		(0.0636)
Annualized population change in 2004-2006		0.00325		
		(0.00696)		
Annualized per capita income change in 2004-2006		-0.0346		
		(0.0286)		
Annualized population change in 2007-2009				-0.00254
				(0.00954)
Annualized per capita income change in 2007-2009				0.0171
				(0.0431)
Constant	0.0677***	0.0297**	-0.118***	0.0150
	(0.00700)	(0.0119)	(0.00907)	(0.0153)
Observations	326	322	326	322
R-squared	0.146	0.209	0.160	0.476

Table 2: Change in the Fraction of Non-Owner-Occupied Home Purchases from 2001-2003 to 2004-2006

This table presents coefficient estimates from regressing the fraction of non-owner occupied home purchases from 2001-2003 to 2004-2006 on the dummies indicating whether a county is in the middle-elasticity group or the elastic group, with the inelastic group as the benchmark and a list of control variables. Robust standard errors are in parentheses. ***, **, * indicate coefficient estimates statistically distinct from 0 at the 1%, 5%, and 10% levels, respectively.

	(1)	(2)
	Change in the fraction of non-owner occupied home purchases from 2001-03 to 2004-06	
Middle group dummy	0.0249***	0.0242***
	(0.00424)	(0.00432)
Elastic group dummy	0.0145***	0.0124***
	(0.00348)	(0.00389)
Fraction of subprime households in 2005		-0.0318
		(0.0237)
Annualized population change in 2004-2006		0.00647
		(0.00444)
Annualized per capita income change in 2004-2006		-0.0385**
		(0.0173)
Constant	0.0190***	0.0257***
	(0.00283)	(0.00593)
Observations	323	319
R-squared	0.071	0.093

Table 3: Change in the Fraction of Non-Owner-Occupied Home Purchases from 2001-2003 to 2004-2006 and the Recent Housing Cycle

This table presents coefficient estimates from regressing the change in real house price in 2004-2006 (housing boom period) and in 2007-2009 (housing bust period) on the change of fraction of non-owner occupied home purchases from 2001-2003 to 2004-2006 and a list of control variables. Robust standard errors are in parentheses. ***, **, * indicate coefficient estimates statistically distinct from 0 at the 1%, 5%, and 10% levels, respectively.

	(1)	(2)	(3)	(4)
	Annualized real house price change in 2004-2006		Annualized real house price change in 2007-2009	
Change in the fraction of non-owner occupied	0.636***	0.681***	-0.494***	-0.628***
home purchases from 2001-2003 to 2004-2006	(0.115)	(0.120)	(0.165)	(0.138)
Annualized population change		-0.00202		
in 2004-2006		(0.00772)		
Annualized per capita income		0.0136		
in 2004-2006		(0.0299)		
Annualized population change				0.00791
in 2007-2009				(0.00991)
Annualized per capita income change				-0.0593
in 2007-2009				(0.0442)
Fraction of subprime households in 2005		0.241***		-0.679***
		(0.0429)		(0.0550)
Constant	0.0336***	-0.00717	-0.0704***	0.0432***
	(0.00475)	(0.00830)	(0.00693)	(0.00951)
Observations	323	319	323	319
R-squared	0.116	0.227	0.034	0.446