

Market Making Obligations and Firm Value*

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Abstract:

We model a contract by which a firm engages a Designated Market Maker (DMM) to provide more liquidity than would be supplied in a competitive market. The DMM contract increases trading volume, and enhances allocative efficiency, price discovery and firm value. The model predicts that DMM contracts will be most valuable for (i) firms characterized by substantial information asymmetries as well as greater uncertainty regarding asset value, and (ii) firms whose real investment decisions are better informed by improved price discovery, i.e. small, young, growth firms. Our analysis indicates that contracts between listed firms and DMMs, which are currently prohibited in the U.S., can represent a market solution to a market imperfection.

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1. Introduction

Recent turmoil, including the financial crisis of 2008-2009 and the “flash crash” of May 6, 2010, highlight the importance of liquidity in financial markets. Modern stock markets largely rely on limit order submissions for liquidity provision. There are no barriers to entry, and essentially any investor can supply liquidity in electronic limit order markets. Nevertheless, in practice liquidity has increasingly been supplied by specialized “high frequency trading” firms, who use computer algorithms to quickly submit and modify large quantities of limit orders.¹ In contrast to the designated “specialists” who in past decades coordinated trading on the flagship New York Stock Exchange, high frequency traders are typically under no obligation to supply liquidity.

The desirability of purely endogenous liquidity provision has recently been questioned, particularly in the wake of the sharp, albeit brief, decline in U.S. equity prices on May 6, 2010. Kirilenko, Kyle, Samadi, and Tuzan (2011) conclude that high-frequency firms did not trigger the “flash crash” on that date, but may have exacerbated it by demanding rather than supplying liquidity after prices started to fall. Other commentators, e.g. Arnuk, Saluzzi, and Leuchtkafer (2011) assert that numerous high frequency firms simply “turned their algo-bots off and disappeared” from the market during the turbulence. Mary L. Shapiro, chairman of the United States Securities and Exchange Commission (SEC), highlighted the issue as follows:

“May 6 was clearly a market failure, and it brought to the fore concerns about our equity market structure.... Where were the high frequency trading firms that typically dominate liquidity provision in those stocks? The issue is whether the firms that effectively act as market makers during normal times should have any obligation to support the market in reasonable ways in tough times.”²

¹ Brogaard (2010), for example, shows that a small group of high frequency firms participate in about two thirds of all trades on the NASDAQ market, while following liquidity-providing strategies. Menkveld (2012) asserts that high frequency traders look “much like an electronic version of the classic market maker.”

² From a speech to the Economic Club of New York, September 7, 2010. Available at <http://www.sec.gov/news/speech/2010/spch090710mls.htm>.

In this paper, we introduce a model that assesses the costs and benefits of contracts that require liquidity providers to supply more liquidity than they would endogenously choose. Contracts that require one or more “Designated Market Makers” (DMMs) to provide liquidity are observed on some markets, particularly for less liquid stocks.³ The most frequently observed obligation is a “maximum spread” rule, which requires the DMM to keep the “bid-ask spread” (the difference between the lowest price for an unexecuted sell order and the highest price for an unexecuted buy order) within a specified width.⁴

Our goal is to develop in the simplest possible model a framework for understanding the circumstances under which DMM contracts can enhance trader welfare and firm value, the economic mechanisms by which the contracts operate, and to assess factors that affect optimal contracted spread widths.⁵ The model includes three periods. At $t=0$, the firm sells the rights to the risky cash flows from its existing assets to investors in an IPO. Investors may subsequently become privately informed regarding the value of the assets, and are subject to a liquidity shock. At $t=1$ secondary trading can occur, potentially motivated by either information or liquidity needs, either in a competitive market for liquidity or in a market with a DMM contract. At $t=2$ the investor receives the value of the asset in cash.

As a benchmark, we determine the IPO price that will just induce investors to participate in the IPO, given competitive liquidity provision in the secondary market, and given the alternative of investing in a completely liquid treasury security. We then assess the effects of a contract by which the firm requires liquidity providers to narrow the secondary market bid-ask spread beyond competitive

³ Charitou and Panayides (2009) report that DMM contracts are employed for at least some stocks in the largest stock markets in Canada, the UK, Germany, France, the Netherlands, Sweden, Spain, Italy, Greece, Denmark, Austria, Finland, Norway, and Switzerland.

⁴ Such obligations typically bind. For example, Anand, Tanggaard, and Weaver (2009) study DMM agreements on the Stockholm Stock Exchange and document that the contracted maximum spreads are typically narrower than the average spread that prevailed prior to the introduction of DMMs. On the New York Stock Exchange, Rule 104 recently introduced, for a set of pilot securities, DMMs who are tasked with maintaining quotes that match the best existing quotes a specified percentage of each trading day. However, Rule 104 does not require the NYSE DMM to improve on the best quotes at any time.

⁵ The answer to the question of why an affirmative obligation to enhance liquidity supply might be desirable cannot simply be “because liquidity is valuable.” A requirement to supply extra liquidity in an otherwise competitive market for liquidity provision will impose trading losses on the DMMs, for which they must be compensated. The contract can be valuing enhancing only if the benefits to listed firms and/or traders from improved liquidity exceed the costs imposed on liquidity suppliers.

levels. Such a contract has two effects. First, it imposes expected trading losses on market makers. Second, it increases the equilibrium IPO price, as investors take into account the benefits of the being able to subsequently trade at lower cost. Notably, the increase in the IPO price can exceed the requisite payment to the market makers, thereby increasing the firm's net proceeds.

Our analysis shows that the narrowing of bid-ask spreads due to a maximum spread rule leads to increased secondary market trading, and improves economic efficiency. In our model, as in the classic analysis of Glosten and Milgrom (1985), the bid-ask spread arises due to information asymmetry. A key point of perspective is that, while informational losses comprise a private cost to liquidity providers, these are zero-sum transfers rather than a cost when aggregated across all agents. Competitive spreads compensate liquidity suppliers for their private losses to better informed traders, and are therefore wider than the net social cost of completing trades. A maximum spread rule can improve social welfare and firm value because more investors will choose to trade when the spread is narrower. This increased trading can enhance allocative efficiency, as traders who value the asset highly acquire more of it, while traders who no longer value the asset highly are able to exit their positions.

However, we find that the constrained spread that maximizes the firm's net proceeds (IPO price less payment to market maker) is typically not zero, even though the only cost borne by liquidity suppliers in our model is that due to asymmetric information. This result reflects the complex role of informed trading in market equilibrium. A narrower bid-ask spread induces more trading. However, trades motivated by information, while privately optimal, can impose their own externality by reducing allocative efficiency.⁶ In cases where the probability of informed trading is sufficiently high it is possible to reduce rather than increase the firm's net proceeds by constraining the spread to be too narrow. Our model predicts that contracts that constrain the spread will be beneficial for firms with

⁶ Hollifield, Miller, Sandas, and Slive (2007) also observe that the allocative efficiency of a market can be reduced by the presence of informed investors who transact based on the divergence of prices from value.

greater relative uncertainty regarding asset value, and where information asymmetry between market makers and investors exists, but is not excessively high.

Narrowing the bid-ask spread has a second potential benefit, in the form of enhanced price discovery. In particular, we show that time 1 trade price becomes a better indicator of security liquidation value (as measured by a lower mean squared forecast error) as the spread is narrowed. More rapid price discovery provides superior information for real decisions, and can lead to increased firm value and improved economic efficiency, as shown for example by Tetlock and Hahn (2007), Foucault and Fresard (2012), Holmstrom and Tirole (1993), and Subrahmanyam and Titman (1999). The model therefore also implies that DMM contracts are more likely to be adopted by firms that benefit more from enhanced price discovery to guide real investment decisions. These will likely be firms with large potential future investments, as opposed to firms in stable, mature, industries.

Our analysis has implications that differ in an important but subtle way from the simple wisdom that DMMs are most useful in otherwise illiquid stocks. If these stocks have wide bid-ask spreads because of high real costs of competing trades, e.g. due to the inventory costs that Demsetz (1968) predicts will be high for thinly-traded assets, then the marginal social cost of providing liquidity is high, and it is efficient for spreads to be wide. In contrast, our analysis focuses on spreads attributable to information asymmetry, and shows that DMM contracts can enhance value as long as the probability of informed trading is not excessive.

Competitive bid-ask spreads will widen at those times and for those stocks where liquidity suppliers perceive an increased likelihood of information-based trading. Easley, Lopez de Prado, and O'Hara (2011) argue that their empirical measure of information asymmetry increased prior to the "flash crash" of May 6, 2010, and that high frequency traders withdrew liquidity in response. Our analysis implies that the resulting reduction in liquidity was economically inefficient, and that future flash crashes

can be potentially be avoided and firm values enhanced by contracts calling for one or more DMMs to continue to provide liquidity during periods of heightened information asymmetries.

The model presented here has direct regulatory implications. We show that contracts by which a firm compensates a DMM for enhancing liquidity provision in its own shares can enhance market quality, trader welfare, and firm value. However, such contracts impose trading losses on, and therefore require side payments to, the DMM. A regulatory requirement that certain DMMs provide additional liquidity beyond competitive levels without such compensation would lead to exit from the industry and would ultimately be counterproductive to the goal of enhancing liquidity.

Contracts of the type we assess are in fact observed on some markets, but are currently prohibited in the U.S. by FINRA rule 5250, which “prohibits any payments by an issuer or an issuer’s affiliates and promoters, directly or indirectly for publishing a quotation, acting as a market maker or submitting an application in connection therewith.”⁷ The NYSE and Nasdaq markets have both recently requested partial exemptions from rule 5250, to allow DMM contracts for certain Exchange Traded Funds.⁸ Some commentators have criticized these proposals on the grounds that DMM contracts “distort market forces.”⁹ Our model shows that the competitive bid-ask spread fails to maximize trader welfare or firm value, due to externalities associated with information asymmetries. DMM contracts of the type considered here comprise a potential market solution to a market imperfection.

2. Related Literature

Many authors provide models of market making. Among these, Demsetz (1968) shows that market maker spreads will decline as a function of typical trading activity in the stock. Ho and Stoll

⁷ FINRA Regulatory Notice 09-60. See Venkataraman and Waisburd (2007) and Anand, Tanggaard, and Weaver (2009) for descriptions of DMM agreements on Euronext Paris and the Stockholm Stock Exchange, respectively.

⁸ See SEC Release No. 34-67411, available at <http://www.sec.gov/rules/sro/nasdaq/2012/34-67411.pdf>. The proposals only call for DMM’s to *match* the best existing quotes at certain times, and thus are less aggressive than the “maximum spread” obligations considered here.

⁹ SEC Release No. 34-67411, page 58.

(1980) assess the effects of inventory on market maker quotes, while Glosten and Milgrom (1985) show that a bid-ask spread will arise if some traders are better informed than liquidity suppliers. Biais (1993) compares liquidity provision in a centralized limit order market to that in a fragmented dealer market. Goettler, Parlour, and Rajan (2005) and Foucault, Kadan, and Kandel (2005), among others, model liquidity provision strategies used by limit order traders. Biais, Foucault, and Moinas (2012) rely on a model that is similar to ours in several dimensions, including a single round of trading, binomial outcomes on fundamental asset value, potentially informed traders, and exogenous liquidity shocks. They focus on market making by well-informed high frequency traders, showing that such high frequency participation enhances economic efficiency by allowing more trades to occur, even while negative externalities are imposed on other informed traders.

However, in the literature cited above, as in most of the literature on market making, the emphasis is on endogenous liquidity provision, i.e. on dealer and trader behavior in the absence of any specific obligation to supply liquidity. Only a few authors have assessed the role of DMMs, and only in limited ways. Venkataraman and Waisburd (2007) consider the effect of a DMM in a periodic auction market characterized by a finite number of investors. The DMM in their model is essentially an additional trader who is present in every round of trading, thereby improving risk sharing. Sabourin (2006) presents a model where a designated market maker is imposed in an imperfectly competitive limit order market. In her model, the presence of a designated market maker will cause some limit order traders to substitute to market orders, which reduces competition in liquidity supply and allows the possibility of wider spreads with a designated market maker.

A small group of empirical researchers have studied the effects of DMMs on market quality. Anand and Weaver (2006) examine the Chicago Board Options Exchange (CBOE) when that market began to assign DMMs to each option, documenting decreased bid-ask spreads and increased CBOE market share following the introduction of DMMs. Petrella and Nimalendran (2003) study the Italian Stock Exchange, documenting improved market quality on a hybrid market that includes a designated

market maker, as compared to stocks traded on a pure limit order market. Venkataraman and Waisburd (2007), Anand, Tanggaard, and Weaver (2009), Skjeltorp and Odegaard (2011), and Menkveld and Wang (2009) study the introduction of DMMs on Euronext-Paris, the Stockholm Stock Exchange, the Oslo Stock Exchange, and Euronext-Amsterdam, respectively. Each study reports improvements in liquidity associated with DMM introduction, and positive stock valuation effects on announcement of DMM introduction.

While the empirical evidence strongly supports the reasoning that DMMs enhance liquidity and firm values, the evidence does not clarify the source of the value gain. Providing enhanced liquidity is costly, and the DMMs must be compensated for these costs. Amihud and Mendelson's (1986) model implies a reduction in firms' cost of capital due to improved liquidity. However, this reasoning alone need not imply that the benefits exceed the costs. Our analysis clarifies the economic mechanism by which value can be enhanced by contracts that narrow bid-ask spreads, and provides insights into the determinants of the optimal contracted spread width.

3. The Model

3.1 Overview

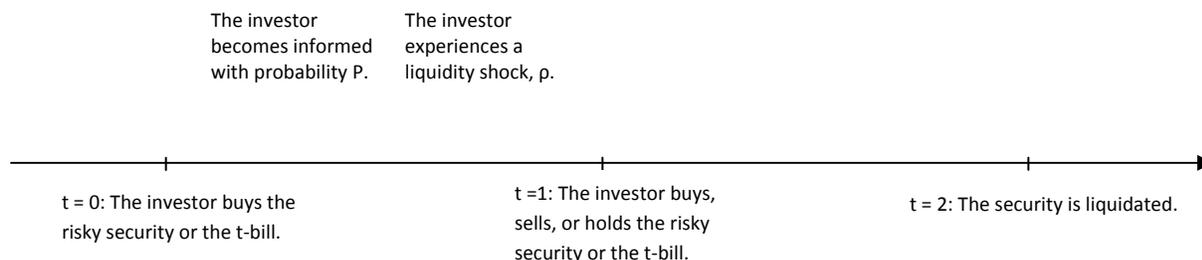
In this section, we introduce a model to assess the potential benefits of a contract between a firm and a DMM. Our intent is to identify the key economic issues with the simplest possible model. An alternative modeling approach would be to consider a contract between the exchange where the firm's shares are listed and a DMM. If the exchange simply recovers the DMM fee from the firm through a higher listing fee, the model remains equivalent to that presented. Alternatively, the exchange might benefit directly, e.g. in the form of trading fee revenue that would be enhanced if trading were increased. "Make-take" fees, which are differential fees assessed on marketable (those that take liquidity) versus non-marketable (those that make liquidity) orders, are employed by a number of exchanges, and subsidize liquidity provision when "make fees" are negative. However, in the absence of frictions Angel, Harris

and Spatt (2011) argue that the net (make – take) will be neutralized in competitive equilibrium by offsetting changes in bid and ask quotes. Foucault, Kadan, and Kandel (2012) present a model where make fees are relevant due to a non-trivial tick size. In contrast, the model we present does not rely on any market friction.

We focus on a single investor who potentially invests in a firm’s initial public offering of stock. In the IPO, the firm sells rights to the eventual cash flow from its risky asset. Proceeds from the IPO, net of any payment to the market maker, are paid to the firm’s existing owners. The IPO investor may want to trade again, and considers possible future trading costs in assessing the amount she is willing to pay for the firm’s risky equity. The risk-neutral investor chooses between purchasing ownership of the firm’s equity and an alternative investment that is free of risk and can be traded without cost.

The model’s timeline is as follows. The firm issues risky equity at $t=0$. The security can be traded in the secondary market at $t=1$, with a liquidity supplier taking the opposite side of the investor’s trade. At $t=2$ the security pays a terminal value, V , equal to either H or L , with equal probability. Let $\mu = (H+L)/2$ denote the unconditional expected payment on the risky security. Prior to $t=1$ trading the investor receives with probability P an information signal that reveals whether security value will be H or L . We consider both the case where P is exogenously determined, and where the investor can endogenously increase P by investing in information acquisition. As an alternative, the investor can purchase a risk-free Treasury bill that makes a certain final payment also equal to μ , and that can be traded without cost.

The timeline is as follows:



As in Glosten and Milgrom (1985), the possibility that the investor is privately informed gives rise to a bid-ask spread in the secondary market for the risky security. To prevent a complete market breakdown requires an extrinsic motive for trade. Just before $t=1$ the investor experiences a liquidity shock, ρ . A positive ρ implies that the investor's subjective valuation of the asset has increased, while a negative ρ implies that the investor's subjective valuation of the asset has decreased. We do not model the determinants of the liquidity shock, which represents *any* motive for trade other than non-public information regarding asset value. For our baseline analysis we assume that ρ is distributed uniform on the interval $-\frac{H-L}{2}$ to $\frac{H-L}{2}$, which allows for a number of closed form solutions. In Section 3.5.1 below we discuss the effects of alternative distributional assumptions regarding ρ .

At $t = 1$ secondary trading occurs, and at $t=2$ the investments pay their liquidation value, either H or L for the risky investment, and μ for the Treasury bill. The risk-neutral investor maximizes expected utility, the ex post outcome on which is the number of units held times the sum of liquidation value and ρ .

We assume the existence of competitive and risk-neutral liquidity providers who have no extrinsic motive for trade ($\rho = 0$ for liquidity providers), and that trades can be completed without any out-of-pocket cost, implying zero-expected profit as the equilibrium condition. This is consistent with the Glosten and Milgrom (1985) model of a dealer market, but also pertains to a limit order market with competitive liquidity provision, which seems a reasonable baseline assumption given a large number of potential limit order traders and the absence of barriers to entry. As in Glosten and Milgrom (1985), the zero profit bid (ask) price is equal to the expected value of the security conditional on observing an investor sell (buy) at $t=1$. The investor's private valuation of the risky equity at $t=1$ is $V^* = \mu + \rho$ if the investor remains uninformed, or $V^* = V + \rho$, where $V = H$ or L , if the investor has become informed. The investor will purchase an additional unit of equity at $t=1$ if V^* exceeds the ask quote, will sell if V^* is less than the bid quote, and otherwise refrains from trading.

3.2 The Treasury-Bill Benchmark.

As an alternative to purchasing the risky security, the investor can purchase one unit of the Treasury bill at $t=0$, for the price μ . The zero-profit condition implies a zero bid-ask spread for the Treasury bill secondary market.¹⁰ The investor will not purchase the risky security unless the expected gain is equal to that of purchasing the Treasury security.

Table 1 summarizes potential outcomes from purchasing the Treasury security at $t=0$. If the investor has a positive subjective valuation shock ($\rho > 0$), she purchases an additional unit at $t=1$ for cost μ . Her ex post gain from being able to purchase an extra unit at time 1 is ρ , and her final subjective valuation for the two units exceeds the 2μ paid to acquire them by 2ρ . If the subjective valuation shock is zero the investor does not trade in the secondary market. The final subjective value of the treasury security equals the amount paid, so the net gain is zero. If the investor has a negative valuation shock ($\rho < 0$) she sells the treasury security in the secondary market for price μ . Since this was also the purchase price, the ex post net gain is zero in this case as well. Note, though, that the ability to sell at $t=1$ has improved the investor's utility, as the net gain would otherwise have been $\rho < 0$.

Table 1: Ex Post Outcomes from Purchasing and Trading the Treasury Security

Time 1 Valuation	If no $t=1$ trade	If trade possible	Trade vs. No-Trade
$V^* = \mu + \rho > \mu \rightarrow$ Buy	$V^* - \mu = \rho$	$2V^* - 2\mu = 2\rho$	P
$V^* = \mu + \rho = \mu \rightarrow$ NT	$V^* - \mu = \rho$	$V^* - \mu = \rho = 0$	0
$V^* = \mu + \rho < \mu \rightarrow$ Sell	$V^* - \mu = \rho$	$\mu - \mu = 0$	$-\rho$

Incorporating the distributional assumption on ρ , the unconditional expectation of the net payoff from buying a Treasury-bill at $t=0$, with the option to trade again in the frictionless market, is:

¹⁰ We assume that the investor cannot short sell the treasury security, or equivalently, that the investor cannot borrow at the risk free interest rate (which is zero in the model). Similarly, the investor cannot sell the risky security unless the time zero IPO takes place, which requires through the participation constraint that the investor be compensated for illiquidity. More generally (as in Glosten and Milgrom, 1985), to prevent a complete market breakdown resulting from information asymmetries, it is necessary that non-information-motivated trade occurs in the secondary market for the risky security.

$$\pi_{tbill} = Prob(\rho > 0) \times E(2\rho | \rho > 0) = \frac{H-L}{4}, \quad (1)$$

which establishes a reservation payoff for the investor.

3.3 The investor's trading strategy and payoff on the risky security

Table 2 summarizes the possible ex post outcomes from participating in the market for the risky equity security, as a function of the investor's subjective valuation, V^* , the bid (B) and ask (A) quotes, the initial offer price, denoted Q , and the possible payment from the firm to the market maker, denoted M . The Table includes outcomes to the investor, the market maker, and the firm, with and without trading at time 1, to emphasize the gains that arise from secondary market trading.¹¹

The first row of Table 2 applies if the investor's subjective valuation at $t=1$ exceeds the ask quote, in which case she purchases another unit in the secondary market. The second row applies if the investor's subjective valuation lies between the bid and ask quotes, and she refrains from trading. The third row applies if the investor's subjective valuation is less than the bid quote, in which case she sells.

In all three cases, the outcome to the firm in the absence of $t=1$ trading, displayed in column (iv) is Q (the proceeds from the IPO) less V (the value of the asset sold). With time 1 trading, outcomes to the firm, displayed in column (v), are Q less the sum of V and M (the payment to the market maker, which equals the market maker's expected trading loss). The firm therefore benefits from the possibility of secondary market trading if the IPO price is increased by more than the payment to the market maker. This payment, M , is determined such that the risk-neutral liquidity supplier has zero expected gain, i.e. so that the unconditional expectation of column (vii) outcomes is zero. In the absence of time 1 trade the investor's ex post outcome is $V^* - Q$. Given the opportunity to trade at $t=1$, the investor's ex post outcome (column ii) is $2V^* - A - Q$ if she elects to purchase a second unit at the ask price, $V^* - Q$ if she refrains from trading, and is $B - Q$ if she elects to sell her share at the bid price.

¹¹ Note that the investor trades with the market maker, not the firm. The market maker's final inventory is therefore the opposite of the $t=1$ investor trade.

Columns (viii) and (ix) reports combined gains, obtained by summing ex post gains across the firm, market maker, and the investor, with and without time 1 trading, respectively. Column (x) reports the change in the combined gain due to time 1 trade. Only the investor's subjective liquidity shock parameter, ρ , appears in these columns. This reflects that all payments in the model are zero-sum across the parties. In particular, while the investor can benefit from trading on private information should she become informed, any investor gain is a market maker loss. Note that the unconditional expectation of the combined gain in the absence of $t=1$ trading (column viii) is zero, which reflects that unconditional mean of the liquidity shock parameter, ρ , is zero. The gain across all parties is positive solely due to time 1 secondary market trading, and can be computed as the unconditional expectation of either column (ix) or column (x).

Table 2: Ex Post Outcomes from Purchasing and Trading the Equity Security

(i)	Investor		Firm		Market Maker		Combined		
	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)
Time 1 Outcome	If no trade	If trade possible	If no trade	If trade possible	If no trade	If trade possible	If no trade	If trade possible	Trade vs. No-Trade
$V^* > A$:	$V^* - Q$	$2V^* - A - Q$	$Q - V$	$Q - V - M$	0	$A - V + M$	ρ	2ρ	ρ
$B < V^* < A$	$V^* - Q$	$V^* - Q$	$Q - V$	$Q - V - M$	0	M	ρ	P	0
$V^* < B$	$V^* - Q$	$B - Q$	$Q - V$	$Q - V - M$	0	$V - B + M$	ρ	0	$-\rho$

From the viewpoint of the investor, the unconditional expectation of the net gain from participating in the IPO (column iii), given $t=1$ trading, can be written as:

$$\begin{aligned} \pi = & Prob(V^* > A)[2E(V^*|V^* > A) - (A + Q)] + Prob(B < V^* < A)[E(V^*|(B < V^* < A)) - Q] \\ & + Prob(V^* < B)[B - Q]. \end{aligned} \quad (2)$$

To induce the investor to participate, the firm chooses Q so that the expected payoff π equals π_{tbill} .

3.4 Model Outcomes

We initially assess the effects of a contract that compensates market makers to narrow the bid-ask spread while assuming that the probability that the equity investor will become informed regarding the $t=2$ asset value is known and exogenous. Proofs are provided in the appendix.

3.4.1 The Competitive Benchmark

Our first results are with regard to bid and ask quotes and the initial offer price, in the absence of a payment from the firm to the market maker. Bid and ask quotes are determined such that the market maker anticipates zero profit conditional on either a customer buy or sell, and the offer price is set to just meet the investor's participation constraint.

LEMMA 1:

For P less than $2/3$, the competitive bid-ask spread is $A-B = \frac{2P(H-L)}{2-P}$, and the firm sets the offer price of the risky security as: $Q^l = \mu - \frac{P(H-L)}{4}$.

For P greater than or equal to $2/3$, the competitive bid-ask spread is $A-B = H-L$ and the firm sets the offer price of the risky security as: $Q^h = \mu - \frac{(2-P)(H-L)}{8}$.

As might be anticipated, e.g. based on the intuition of the model presented by Amihud and Mendelson (1986), the competitive offer price is less than the expected asset value, μ , indicating that the IPO price is discounted to reflect illiquidity in the secondary market, for all $P > 0$. The competitive bid-ask spread increases in the probability of informed trading for $P < 2/3$, and increases in uncertainty regarding final value, $H-L$. The offer price which satisfies the participation constraint decreases with P up to $2/3$, but increases with P for $P > 2/3$. This reflects that higher P benefits the investor, ceteris paribus. However, for $P < 2/3$ the bid-ask spread increases with P , and the wider spread more than offsets the

benefit to the trader. For $P > 2/3$ the competitive spread is already widened to its maximum and the investor strictly benefits from a higher probability of becoming informed.

3.4.2 Spread constraints and net proceeds to the firm.

We now assess the effects of a contract by which the firm requires liquidity providers to narrow the bid-ask spread, and compensates them for doing so. We focus on a contract that requires the bid-ask spread to be no greater than K , and for simplicity focus on symmetric spreads such that $A = \mu + \frac{K}{2}$ and $B = \mu - \frac{K}{2}$.

LEMMA 2: *Given exogenous P and a constrained bid-ask spread K , the equilibrium offer price is:*

$$Q_K = \mu + \frac{P(H-L)}{4} + \frac{(2-P)}{8(H-L)} K^2 - \frac{K}{2}.$$

Note that for any constrained spread, K , the offer price increases as P increases, as $\frac{\partial Q_K}{\partial P} = \frac{(H-L-K)(H-L+K)}{4(H-L)} > 0$. The investor benefits from higher probability of becoming informed, and given a constrained spread, does not pay higher trading costs to offset.

We next turn to the effect of the spread constraint on the firm's offering proceeds.

PROPOSITION 1: *The firm's IPO proceeds always increase as the $t=1$ bid-ask spread is further constrained relative to the competitive spread.*

$$\text{When } P \leq \frac{2}{3}, \quad Q_K - Q = \frac{(2-P)}{8(H-L)} K^2 - \frac{K}{2} + \frac{P(H-L)}{2} > 0$$

$$\text{When } P \geq \frac{2}{3}, \quad Q_K - Q = \frac{(2-P)}{8(H-L)} K^2 - \frac{K}{2} + \frac{(2+P)*(H-L)}{8} > 0$$

Proposition 1 shows that the firm captures benefits from constraining the $t=1$ spread in the form of a higher $t=0$ offer price. However, constrained the spread to be less than the zero-profit spread

necessarily imposes expected trading losses on liquidity providers. To induce the liquidity providers to accept the constraint requires a side payment to compensate for expected trading losses.

LEMMA 3: *For constrained spreads narrower than the competitive benchmark, the liquidity provider's expected market making loss, M , which also equals the requisite payment from the firm to the market makers, depends on the constraint K according to:*

$$M = \frac{(2 - P)}{4(H - L)} K^2 - \frac{(2 + P)}{4} K + \frac{P(H - L)}{2} > 0,$$

and the effect of the spread constraint on market making loss is:

$$\frac{\partial M}{\partial K} = \frac{(H - L)(P + 2) - 2K(2 - P)}{-4(H - L)} < 0.$$

Lemma 3 shows that market maker loss always increases as the contracted spread width is narrowed relative to the competitive spread.

Proposition 1 shows that the firm's IPO price is increased by constraining secondary market spreads, while Lemma 3 demonstrates that the losses imposed on market makers, which must be compensated by the firm, increase as spreads are more constrained. These results are intuitive. The central issue is whether the IPO proceeds are increased by more than the requisite payment to the market makers.

LEMMA 4: *The firm's net proceeds (FNP), defined as the IPO proceeds less the payment to the market maker, depend on the spread constraint, K , according to:*

$$FNP = Q_K - M = -\frac{(2-P)}{8(H-L)} K^2 + \frac{P}{4} K + \frac{2(H+L)-P(H-L)}{4}. \quad (4)$$

We now assess the relation between the constrained spread and the firm's net proceeds.

PROPOSITION 2: *Constraining the spread to an optimal width of $K^* = \frac{P(H-L)}{2-P}$ increases IPO proceeds by more than the required payment to market makers.*

Proposition 2 demonstrates the existence of an optimal spread constraint. Note that the value-maximizing spread is non-zero. When $P < 2/3$, the value maximizing spread K^* is half of the competitive spread. When $P > 2/3$, K^* exceeds half of the competitive spread. Note also that the optimal constrained spread is strictly increasing in uncertainty regarding value ($H-L$), and in the probability of informed trading, P , as displayed on Figure 1.

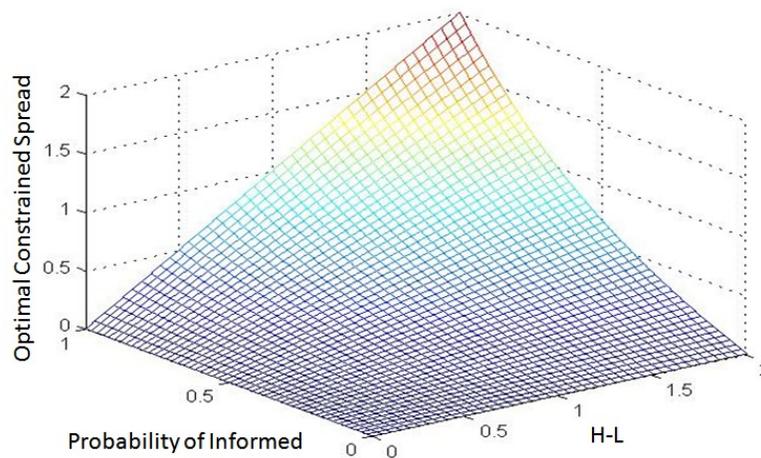


Figure 1 Optimal Constrained Spread for various P and (H-L)

Figure 2 depicts the increase in IPO proceeds, net of the requisite payment to market makers, when the spread is set at K^* , for various levels of P , and given that $H=2$ and $L=1$. Net proceeds are enhanced for all P except $P = 0$ (when the unconstrained spread is zero) and when $P = 1$. The increase in net proceeds is largest for $P = 2/3$.

Proposition 2 comprises the central result of this paper. It implies that firm value is enhanced by a contract requiring liquidity providers to narrow spreads beyond competitive levels, even when the firm compensates liquidity providers for expected trading losses. It also provides the empirically testable

implications that the value-maximizing spread is positive, increases in both the probability of informed trading, P , and increases in the degree of uncertainty regarding asset value, $H-L$.

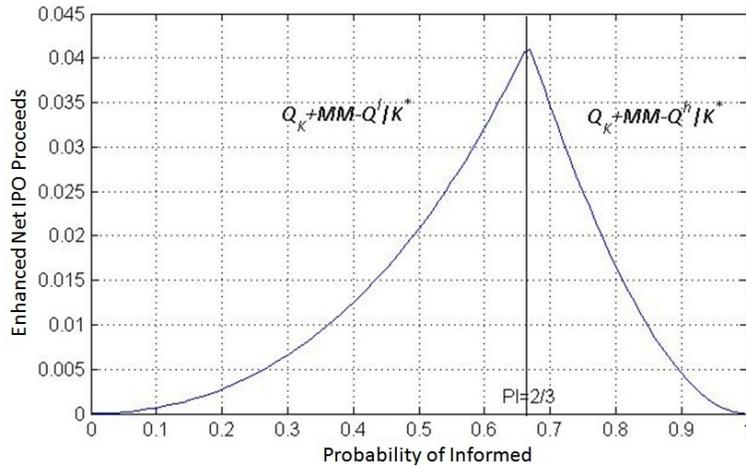


Figure 2 Enhanced Net IPO proceeds with the optimal constrained spread, for various P .

To understand why firm value can be improved by constraining the spread such that liquidity providers suffer trading losses, recognize that bid-ask spreads arise in this model only due to information asymmetries. Competitive liquidity providers charge spreads as a function of their expected losses to better informed traders. Crucially, however, any loss experienced by a liquidity provider is offset by an equal gain to the investor. That is, losses to better informed traders are a private cost to liquidity providers, but are not a net cost when aggregating across all traders. This implies that the competitive bid-ask spread is wider than the net social cost of completing trades.¹²

Since the model incorporates no out-of-pocket costs to completing trades, it is efficient from an allocative perspective for the trader to purchase (sell) any time p is positive (negative). With a positive bid-ask spread the trader is dissuaded from trading in those cases where $B < V^* < A$. The constraint on

¹² Glosten (1989) also observes that a zero-expected-profit condition that applies for each trade leads to inefficiencies when asymmetric information exists. He considers the extent to which a market maker with a degree of monopoly power can improve efficiency, by using profits on some trades to subsidize losses on others. In contrast, we focus on competitive liquidity provision, as in an open limit order book.

spread widths increases allocative efficiency and net IPO proceeds by facilitating efficient trades that would otherwise be dissuaded by a bid-ask spread wider than the social cost of completing trades.

Proposition 2 indicates that net IPO proceeds can be enhanced by optimally constraining the spread. However, narrowing the spread to arbitrary levels has the following implications:

PROPOSITION 3: *When $P < 2/3$, value is enhanced for all positive K narrower than the competitive spread. When $P > 2/3$, value is enhanced when $\frac{(3P-2)(H-L)}{2-P} < K < \frac{2P(H-L)}{2-P}$, but value is decreased when $0 < K \leq \frac{(3P-2)(H-L)}{2-P}$.*

To illustrate Proposition 3, Figure 3 displays the increase in net proceeds for various constrained spreads and P , for $P < 2/3$. Figure 4 displays corresponding changes in net proceeds when $P > 2/3$, and Figure 5 combines the data to display changes in net proceeds from constraining spreads for the full range of P from 0 to 1.

Figures 3 to 5 demonstrate that firm net proceeds are enhanced by constraining the spread for most parameter values. However, two potentially surprising results are also illustrated. First, the spread width that maximizes net proceeds is greater than zero, even though the model incorporates no out-of-pocket cost to complete trades. Second, net proceeds can be decreased rather than enhanced if the spread is constrained too tightly when there is a sufficiently high probability of informed trading, P .

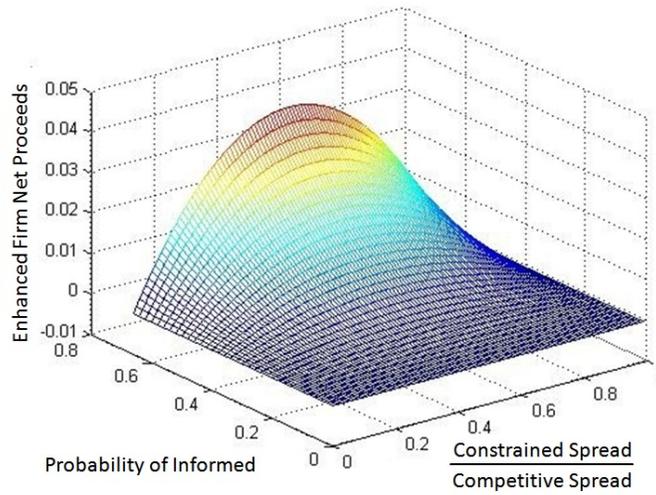


Figure 3 Enhanced Firm Net Proceeds with the constrained spread normalized by the zero-profit competitive spread with P ranging from 0 to $2/3$.

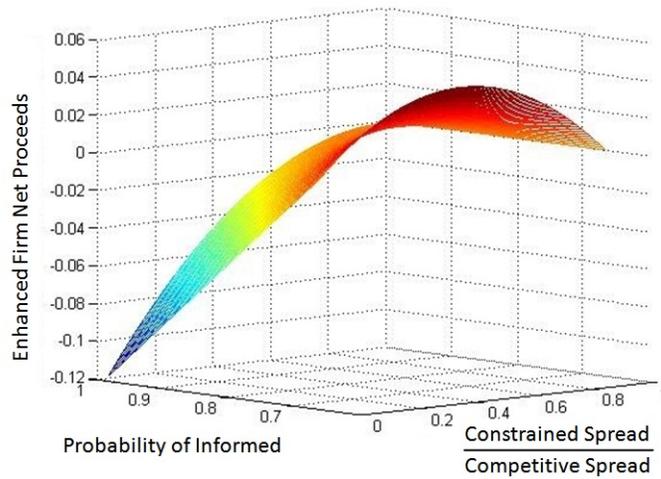


Figure 4 Enhanced Firm Net Proceeds with the constrained spread normalized by the zero-profit competitive spread with P ranging from $2/3$ to 1.

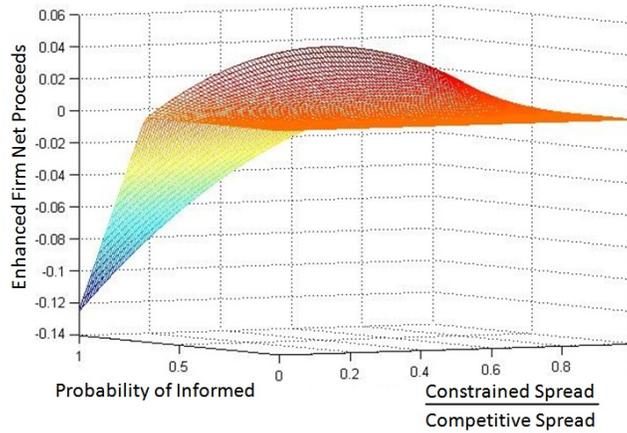


Figure 5 Enhanced Firm Net Proceeds with the constrained spread normalized by the zero-profit competitive spread with P ranging from 0 to 1.

It may be surprising that value can be decreased by the combination of a high probability of informed trading and narrow bid-ask spreads, given that the investor benefits from each. In fact, Lemma 2 above shows that the firm’s IPO proceeds strictly increase with P for any K , and Proposition 1 shows that the firm’s IPO proceeds increase as the spread width is further constrained, for any P . However, to enhance value the gains to the trader must exceed the costs imposed on the market maker, and in some cases they do not.

To understand these results, consider again column (x) of Table 2, which summarizes ex post gains aggregated across the investor, firm, and market maker. These gains depend on the investor’s buy-sell decisions and outcomes on the liquidity shock parameter, ρ . Trading gains across all parties would be maximized if the trader would buy for any realization of $\rho > 0$ and would sell for any realization of $\rho < 0$. Such an outcome would imply perfect allocative efficiency, in the sense that an additional share would flow to the trader if she valued it more highly than the market maker (for whom $\rho = 0$), while the share would be transferred to the market maker if he valued it more highly than the trader.

However, perfect allocative efficiency is not obtained in this setting, due both to the existence of the spread and the fact that trade prices differ from the true asset value known to informed traders.

Constraining the spread potentially improves allocative efficiency by allowing trades to occur that otherwise would not. However, allocative efficiency can be compromised by constraining the spread, by encouraging trades by informed traders in the “wrong” direction.¹³

Referring to column (x) of Table 2, the unconditional expectation of the combined gains from t=1 trading can be expressed as:

Combined Gain from Trading (CGT) =

$$\text{Prob}(V^* > A) * E(\rho|V^* > A) + \text{Prob}(V^* < B) * E(-\rho|V^* < B). \quad (5)$$

Incorporating the distributional assumption on ρ , the unconditional expectation of the combined gains from trade depends on K according to:

$$CGT = \frac{P-2}{8(H-L)} K^2 + \frac{P}{4} K + \frac{(H-L)-P(H-L)}{4}. \quad (6)$$

We can now state the following.

PROPOSITION 4: The spread constraint that maximizes the combined gain from trading is that which maximizes firm net proceeds.

Comparing expression (6) for gains from trade to expression (4) for the firm’s proceeds net of the payment to market makers and expression (1) for the trader’s reservation utility from trading the Treasury security, implies $FNP = \mu + CGT - \pi_{tbill}$. The firm captures as net proceeds the unconditional expected value of the asset, μ , plus the expected combined gains from t=1 trading, less the reservation utility available if the trader purchases the Treasury security instead of the IPO. Since μ and π_{tbill} are not affected by the secondary market spread, FNP and CGT are maximized simultaneously.

¹³ Trades in this model are always privately optimal. Trades in the “wrong” direction occur because the market price of the asset differs from its true, non-public, value. A social planner with the goal of maximizing gains from trade might require all traders to transact in the “right” direction. However, such a requirement would compromise price discovery, would require some form of coercion, and in any case is not induced via a maximum spread rule.

The effect of the spread constraint on the firm's proceeds and gains from trade can be understood by further assessing expression (5). Tightening the spread constraint (decreasing K) decreases the ask and increases the bid. Hence, the probability of a $t=1$ trade strictly increases as the spread is constrained. If the trader is uninformed, $V^* = \mu + \rho$, so V^* exceeds A , giving rise to a buy, if and only if $\rho > 0$, while V^* is less than B , giving rise to a sell, if and only if $\rho < 0$. Narrowing the spread therefore always improves allocative efficiency when the trader is uninformed. In contrast, trades need not improve allocative efficiency when the trader is informed.

For an informed trader, $V^* = H + \rho$ when $V = H$, or $V^* = L + \rho$ when $V = L$. Figure 6 illustrates informed trading outcomes in the former case; outcomes are symmetric when $V = L$. Case 1 illustrated on Figure 6 incorporates the distributional assumption on ρ used to this point, i.e. that ρ is uniformly distributed on the interval $-(H-L)/2$ to $(H-L)/2$. (Cases 2 and 3 will be discussed in Section 3.5.1 below).

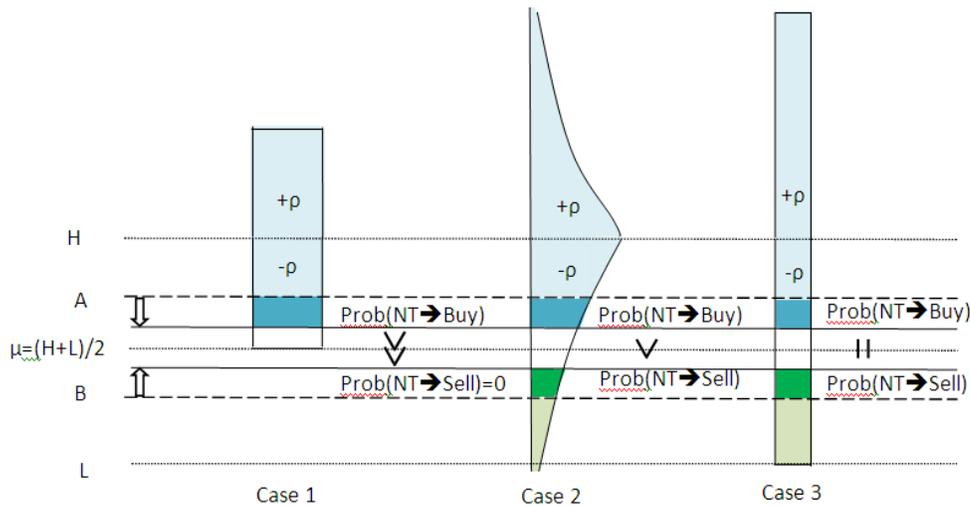


Figure 6: The effect of constraining the spread on informed trading when $V=H$. In Case 1 ρ is uniformly distributed on the interval $-(H-L)/2$ to $(H-L)/2$. In Case 2 ρ is normally distributed with mean zero. In Case 3 ρ is uniformly distributed on the interval $-(H-L)$ to $(H-L)$.

Narrowing the spread decreases A and increases B , each toward μ , as indicated by the arrows on Figure 6. Given $V = H$ and the Case 1 distributional assumption, the lower bound on the informed

trader's subjective value, V^* , is μ . Therefore the decrease in the ask leads to a greater probability of an investor buy, while the increase in the bid does not affect the probability of a sale. However, since $A < H$, the potential realizations on ρ that shift the informed trader from no-trade to buy involve $\rho < 0$. Narrowing the spread therefore increases the probability of an informed trader buy, but decreases the mean ρ conditional on a buy. While the buy induced by a narrower spread is privately optimal for an informed trader (and contributes to price discovery, as discussed in Section 3.4.3), it reduces allocative efficiency, which in turn reduces gains from trade and the firm's net proceeds.

The spread constraint that maximizes the firm's net proceeds is that which maximizes expected allocative efficiency. Constraining the spread to the optimal level always enhances value, but constraining the spread to zero encourages too many informed trades that reduce allocative efficiency. It is possible to reduce rather the net proceeds to the firm if the spread is constrained too far in the presence of a large probability of informed trading.

However, the preceding insight will be at least partially mitigated in practice, for two reasons. First, the available empirical evidence indicates moderate rates of informed trading. In particular, Easley, Kiefer, O'Hara, and Paperman (1996) introduce a model to estimate the probability of informed trading in U.S common stocks, reporting a cross-sectional average P estimate of about 0.20, and that the estimated P is less than 0.40 for virtually all stocks. Our analysis indicates excessively constraining the spread can destroy rather than enhance value only when P approaches one. Further, our results regarding allocative efficiency stem in part from the fact that the model includes just a single round of trading. A more complex model that allows for an offsetting trade after prices adjust to fully incorporate the informed traders' information would alleviate the allocative inefficiency. However, as long as trade prices differ from fundamental value (including the potential effects of subsequent information shocks) the reductions in allocative efficiency due to informed trading will not be completely eliminated, and the optimal spread will not be zero.

3.4.3 Constrained Spreads and Price Discovery

The preceding section shows that optimally constraining the spread leads to an increase in the IPO price that exceeds the cost of reimbursing liquidity providers for their trading losses. This comprises a major benefit from a DMM contract. In addition, the firm potentially benefits from improved price discovery. We focus in particular on the mean squared forecast error (MSFE) when viewing the $t=1$ trade price as a forecast of the $t=2$ liquidation value, H or L . Letting $E_1(V)$ denote the expectation of liquidation value formed conditional on observing the time 1 trades, the MSFE is:

$$MSFE = \text{Prob}(V = H) * [E_1(V|V = H) - H]^2 + \text{Prob}(V = L) * [E_1(V|V = L) - L]^2,$$

where the probabilities are unconditional. In the Appendix we assess these conditional expectations. Let $MSFE^l$ and $MSFE^h$ denote the mean squared forecast error with the competitive spreads, when $P < 2/3$ and when $P \geq 2/3$ respectively. These can be expressed as:

$$MSFE^l = \frac{(H-L)^2(-4P^3+3P^2+4P-4)^2}{4(2-P)^4}, \text{ and } MSFE^h = \frac{(H-L)^2(2-P)^2}{16}.$$

Let $MSFE_K$ denote the mean squared forecast error when the spread width is constrained to K . We have:

$$MSFE_K = \frac{(P^2K^2+2(H-L)(2-P-2P^2)K+4P^2(H-L)^2)^2}{16(2H-2L-2K+P*K)^2}.$$

PROPOSITION 5: *Constraining the bid-ask spread improves price discovery, as the mean square forecast error (MSFE) of the time 1 trade price as a predictor of time 2 value is reduced.*

Figure 7 displays the difference between the MSFE with various constrained spreads and the MSFE with the zero-profit competitive spread for various constraints and probabilities of informed trading, and shows that the MSFE declines as the spread is constrained, for K and all P . The forecast error is reduced most when P is large, which reflects that the improvement in price discovery arises due to a narrower spread encouraging informed traders to trade on their information.

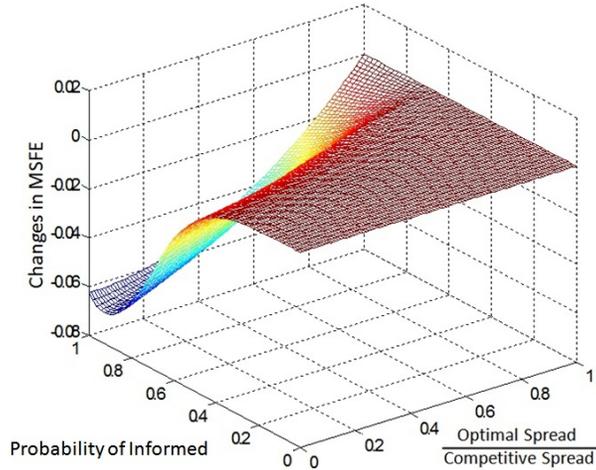


Figure 7: The effect of the constrained spread on the mean square forecast error, relative to the competitive benchmark.

This analysis highlights the complex role of informed trading in security market equilibrium. Narrower spreads induce more outcomes where the informed trader is willing to purchase because $V=H$ even though her liquidity shock is negative, and more outcomes where she is willing to sell because $V=L$, even though her liquidity shock is positive. These additional trades are informative, and enhance price discovery. On the other hand, the same trades reduce allocative efficiency, as discussed in the preceding section.

3.5 Extensions

We have shown that a contract between the firm and a DMM that calls for narrowing the secondary market bid-ask spread to less than competitive levels can benefit the firm in two ways. First, it can increase the market value of the firm's securities by more than the required payment to the liquidity provider. Second, by encouraging informed traders to transact on their information, it can improve price discovery, such that the secondary market transaction price becomes a more reliable forecast of the final value of the firm's assets.

However, our analysis to this point has relied on a particular distributional assumption regarding liquidity shocks, and on the assumption that the probability that the trader will become privately informed, P , is exogenous. Further, while the distributional assumption used allows for closed form solutions, it allows for only limited insights regarding cross-sectional variation in the optimality of spread constraints. We now consider the effects of relaxing these assumptions in various dimensions.

3.5.1 Alternative Distributional Assumptions

The results presented in Section 3.4 relied on a specific distributional assumption regarding the investor's subjective motive for trade, in particular that ρ is uniformly distributed on the interval $-(H-L)/2$ to $(H-L)/2$. This assumption is convenient, and allows for a number of closed form solutions. However, the actual distribution of motives for trade is unknown, and a wide variety of alternative assumptions could potentially be evaluated. Here, we briefly outline two alternatives that highlight the key features of possible alternative distributions, and argue that our central results are likely to be robust to reasonable alternative specifications.

If the trader is uninformed, then for any symmetric distribution of ρ centered on zero, narrowing the spread will lead to more uninformed buys with positive ρ and more uninformed sells with negative ρ . For any such symmetric distribution narrower spreads lead to more uninformed trades. This necessarily improves allocative efficiency, but also generates more noise for purposes of price discovery. Similarly, narrower spreads will always generate more trades by informed traders, for any symmetric distribution of ρ centered on zero. However, the increased informed trades do not necessarily improve allocative efficiency, and may or may not increase price discovery, depending on the distribution of ρ .

Figure 6 illustrates outcomes on V^* , the informed investor's subjective valuation of the security, and the informed trader's trading decisions for two additional distributional assumptions. In Case 2, ρ is distributed normally with mean zero. In Case 3, ρ is uniformly distributed on the interval $-(H-L)$ to $(H-L)$. Figure 6 focuses on the case where $V = H$; results when $V = L$ are symmetric.

The relevant attribute of the two new probability distributions, relative to the base case assumption, is that they each allow for outcomes on V^* (conditional on $V = H$) that are less than the unconditional expected asset value, μ , while the base case did not. As a consequence, narrowing the bid-ask spread can lead to additional informed sales as well as purchases in Cases 2 and 3, while in the base case narrowing the spread could only lead to additional informed purchases. In Case 2, outcomes on V^* that are less than H but exceed μ are more likely than outcomes that are less than μ , implying that a symmetric narrowing of the spread will induce more additional purchases than sales. In case 3 outcomes on V^* that are less than H but exceed μ are equally likely as outcomes that are less than μ , implying that a symmetric narrowing of the spread will generate an equal number of additional sales as additional purchases.

The key attribute of case 3 is that narrowing the spread is equally likely to generate additional informed sales as additional purchases. In contrast, under the base case assumption constraining the spread was more likely to generate additional buys when value is high and more sells when value is low. In case 3, price discovery is not enhanced by constraining the spread, because an equal number of new buys and new sells are induced by the narrower spread, rendering the information content of buys versus sells unchanged. By comparison, the improvement in price discovery in the base case occurs because constraining the spread was more likely to induce customer buys (sells) when value is high (low). Further, in case 3, the constrained spread that maximizes the firm net proceeds is zero.¹⁴ This result also reflects that constraining the spread is equally likely to induce new buys as to induce new sells in case 3. As a consequence, allocative efficiency in trades stemming from informed traders does not decrease on average with a narrower spread in case 3, as it did under the base case assumptions. The improved allocative efficiency associated with increased trades by uninformed traders therefore leads to a value-maximizing spread of zero in case 3.

¹⁴ Formal proofs of both Case 3 results cited in this paragraph are available from the authors on request.

Comparing case 3 to base case highlights that a key feature of any possible distributional assumption regarding liquidity shocks is whether small shocks (in particular those with absolute value less than $(H-L)/2$) are more likely than large shocks (in particular those with absolute value greater $(H-L)/2$). The base case involves zero probability of large shocks, while case 3 involves equal probabilities of large and small shocks. While the actual distribution of liquidity shocks is not observable, we consider probability distributions where small shocks are more likely than large to be more realistic. This analysis indicates that our results that price discovery is improved by narrowing the spread and that the optimal spread is non-zero can be overturned if large liquidity shocks are assumed to be as likely as small shocks, but that the results are likely to be robust to other alternative distributional assumptions. Note, though, that the key result that value is enhanced by constraining the spread survives even in case 3.

Normally distributed shocks, as in case 2, comprise a particular intermediate assumption. Narrowing the spread (still conditional on $V=H$) increases sales by informed traders, but to a lesser extent than it increases purchases by informed traders. As a consequence, narrowing the spread in Case 2 unambiguously improves price discovery, as it did under base case assumptions.

Whether the constrained spread that maximizes firm net proceeds in Case 2 remains positive or is equal to zero depends also on the standard deviation of liquidity shocks. Narrowing the spread induces more purchases by informed traders with negative ρ than sales by informed traders with positive ρ , but the latter involve liquidity shocks of greater absolute value. In the following section we implement numerical searches and find optimal spreads that are non-zero, given normally distributed liquidity shocks.

3.5.2 Cross-Sectional Variation in the Use of DMM Contracts

As noted, DMM contracts are used on a number of international stock markets. Such contracts are typically not used for all stocks on the market, but tend to be concentrated in less liquid securities. Our analysis shows that the key characteristic that allows a DMM contract to improve value is not

illiquidity per se, but illiquidity attributable to information asymmetry. However, the analysis in Section 3.4 does not clarify why some firms refrain from entering DMM contracts.

To assess cross-sectional variation in the usage of DMM contracts requires alternative assumptions regarding the distribution of liquidity shocks, ρ . In our baseline analysis, the distribution of ρ is linked to the fundamental uncertainty in asset value, $H-L$, which allowed for closed-form solutions that helped to illustrate the key economic reasons that DMM contracts can enhance value. However, to obtain cross-sectional insights requires assumptions that break this linkage, as fundamental uncertainty is likely to vary across assets, while the distribution of liquidity shocks is more appropriately viewed as an investor characteristic that remains the same regardless of which asset is purchased.¹⁵

To assess cross-sectional variation in DMM contract usage, we focus on a measure of relative uncertainty regarding asset value, $R = (H-L)/(H+L) = (H-L)/2\mu$, which is absolute uncertainty scaled by expected asset value. Higher relative uncertainty would generally be associated with smaller, younger, and more volatile firms. While uncertainty regarding asset value differs across securities, we assume that liquidity shocks are drawn from the same distribution, regardless of the asset purchase. We focus in particular on liquidity shocks drawn from a zero-mean normal distribution.¹⁶

Unlike the baseline analysis, where the expected payoff on both the risky and riskless securities equal μ , the cross-sectional analysis includes risky investments that differ in scale. This gives rise to a modified indifference condition for IPO participation. In particular, the expected utility gain per unit of μ must be equal across all risky assets and equal to that of the T-bill. We continue to rely on expression (2) to describe investor utility, and appendix expressions A.III.1 and A.III.2 to define competitive equilibrium. However, closed form solutions for the equilibrium offer price, the required payment to

¹⁵ Of course, in a more complex model with heterogeneous investors who differ in terms of the distribution of liquidity shocks it is possible that clientele effects could form.

¹⁶ Results are based on a standard deviation of liquidity shocks equal to one. More generally, model results depend on relative uncertainty in asset value as compared to the standard deviation of liquidity shocks. We rely on the normal distribution here since closed form solutions are not available in any case, and a distribution where small liquidity shocks are more common than large seems more realistic as compared to the uniform distribution.

market makers and firm net proceeds are not available when the distribution of ρ is no longer linked to absolute uncertainty or when ρ is distributed normal rather than uniform. We therefore implement numerical procedures, searching across constrained spreads ranging from the competitive spread to zero, in order to assess optimal firm-specific spread widths.

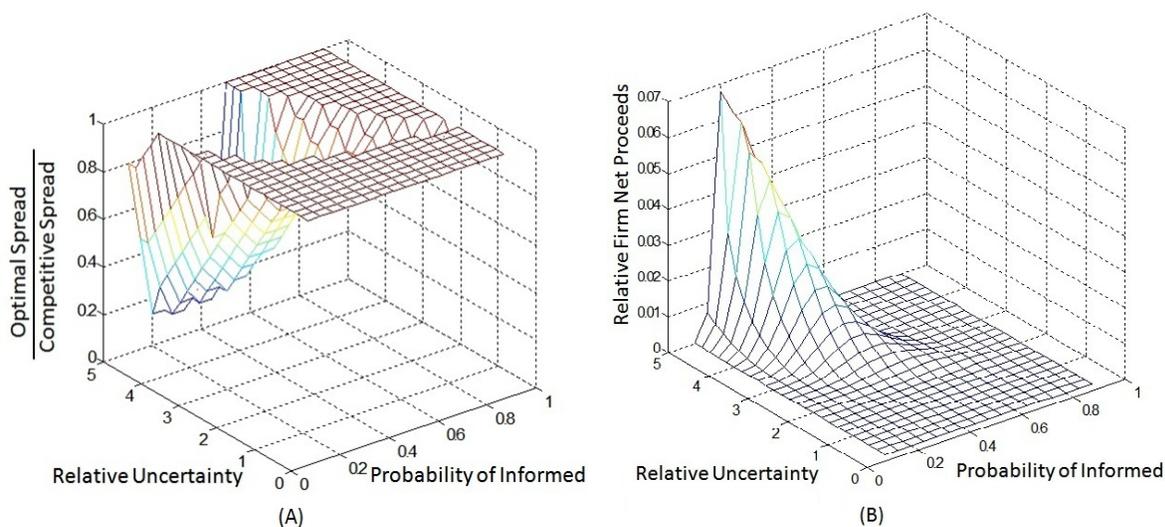


Figure 8: (Panel A) Displays optimal spreads for various probabilities of informed trading, P , and relative uncertainty regarding asset value, R . Panel (B) displays the change in relative firm net proceeds for various probabilities of informed trading, P , and relative uncertainty regarding asset value, R .

Figures 8A and 8B display the optimal constrained spread (relative to the zero profit spread) and the enhancement in firm net proceeds as a result of constraining the spread, for various probabilities of informed trading, P , and relative uncertainty regarding asset value, R . The figures show that the optimal constrained spread is less than the competitive spread, and firm net proceeds are enhanced by constraining the spread, when relative uncertainty is high, and when the probability of informed trading is moderate. This result reflects the fact that the competitive spread itself is wide for firms with high relative uncertainty, meaning that a higher proportion of traders are dissuaded from trading. Narrowing the spread induces proportionately more trading by uninformed traders, whose trades always improve

allocative efficiency, when the spread is wider. This leads to improved allocative efficiency for firms with higher relative uncertainty and higher competitive spreads, unless the probability of informed trading is excessive.

As noted, Easley, Kiefer, O'Hara, and Paperman (1996) report empirical estimates of informed trading probabilities that are less than 0.40 for virtually all stocks. These moderate estimates of P , in combination with our model results, imply that DMM contracts will indeed enhance value, for firms with large relative uncertainty in asset value. These will tend to be smaller and younger firms. This model implication accords with simple intuition and the empirical observation that DMM contracts tend to be employed for less liquid firms (which tend to be younger and smaller). However, we emphasize that the contracts enhance value by reducing externalities stemming from information asymmetries, not due to illiquidity attributable to non-informational sources.

3.5.3 Model Outcomes with Endogenous Information Acquisition

In section 3.4, we show that given an exogenous probability that the trader becomes informed, firm net proceeds and price discovery can both be enhanced by constraining the bid-ask spread to be narrower than the zero-profit spread. We now relax the assumption of exogenous P , to reflect that traders can choose to invest in information. We return in this analysis to the base case assumption that liquidity shocks are distributed uniform.

We assume that P can be increased as the trader invests in information acquisition, according to a convex cost function $\eta(P) = \frac{P \times c}{1-P}$, where c is a constant. This implies that investors can improve the odds of becoming informed regarding true value, but that the cost of doing so increases as more certainty is sought. After paying η , the investor will learn the true $t=2$ value of the security with probability P .

The investor benefits directly from higher P through better trading decisions. In addition, we assume that the value of the firm's assets is enhanced when P is increased, according to the concave

function $\lambda(P) = b\sqrt{P}$, where b is a constant. We do not model of the effect of P on the value of the firm's assets, as this effect has been addressed in the extant literature, and our primary goal lies in assessing the effect of contracts that enhance liquidity supply. The basic rationale is that, given more informative secondary market prices, the firm can make superior investment and compensation decisions, as in Foucault and Fresard (2012), Tetlock and Hahn (2007), Holmstrom and Tirole (1993), Dow and Gorton (1997), and Subrahmanyam and Titman (1999), among others.

The investor gains this benefit through ownership of the firm's equity, implying that the investor's expected time zero payoff from initially investing in the risky security becomes $\pi' = \pi + \lambda - \eta$. Note though, that since the IPO price is set to provide the investor with only her reservation utility, the firm's current owners ultimately capture all benefits (net of η) from improved price discovery, in the form of a higher IPO price.

For any given spread, the investor evaluates marginal costs and benefits to choose her optimal P , which we denote P^* . Since liquidity providers know the investor's costs and benefits, they can infer P^* . In the absence of any constraint on spreads, liquidity suppliers continue to choose zero-profit bid and ask quotes, on the basis of P^* . Similarly, the firm knows P^* , and selects the offer price Q so that the participation constraint is met.

The investor pays the security offer price, Q , before choosing P , implying that the offer price is a sunk cost at the time of the decision to invest in information. If the firm is committed to constraining the spread the investor knows the contracted spread width, K , at the time she chooses P . This creates an important asymmetry between constrained and unconstrained outcomes. In the absence of a constraint, increasing P leads to wider spreads, and the investor takes this effect into account. However, with fixed K , increasing P does not widen spreads; it instead increases the side payment from the firm to the liquidity provider. The trader therefore has stronger incentives to invest in increasing P with contractually fixed as compared to competitive spreads.

Let $P^*(K)$ denote the trader's optimal P as a function of the constrained spread. The corresponding payoff to the investor can be denoted $\pi'(K, Q, P^*(K))$. Equilibrium requires $\frac{\partial \pi'}{\partial P} = 0$ and $Q = \pi' - \frac{(H-L)}{4}$, where the former is the first order condition that reflects the informed trader's optimal choice of P , and the latter is the investor's participation constraint. Unfortunately, these equilibrium expressions cannot be solved analytically.

We instead assess numerical solutions. Figure 9 displays the gross investor payoff, π' , for various P and K , based on parameters of $H=2$, $L=1$, $b=0.36$, and $c=0.03$. The optimal P that will be selected by the investor for any given K chosen by the firm can be assessed as the highest outcome on π' for the given K . As Figure 9 demonstrates, the investor will select a higher P when the spread constraint, K , is tighter.

Figure 10 displays with a solid blue line firm net proceeds (FNP) for various levels of the constrained spread, K , conditional on the investor's optimal selection of $P^*(K)$. As when P was fixed, there is an interior optimum K^* that maximizes firm net proceeds. Figure 10 also displays, with the dashed green line, the optimal P^* that the investor would choose for different levels of the spread constraint, K . As might be anticipated, P^* decreases with K , because the marginal gain from being informed is increased if the cost of trading on the information is reduced. However, the modest increase in P^* , from about .785 when $K = H-L$ to about .805 when $K = 0$ understates the importance of constraining the spread, relative to the outcome when the spread is endogenous.

Figure 11 displays the sum of firm net proceeds, FNP, and fixed reservation utility, with the competitive spread, for various possible P . Since the firm makes no payment to the market maker in this case, the firm's net proceeds plus reservation utility is simply the investor's expected trading gains. The investor chooses P after paying the IPO price, so as to maximize this net gain. Note that, in the absence of a payment to market makers, the competitive spread will increase if the trader invests to increase P . This creates a disincentive to increase P that is absent for any fixed K . As Figure 11 shows, FNP plus reservation utility is maximized at $P = 0$. That is, with competitive market making (and given the selected

parameters) the marginal costs of becoming informed exceed the marginal benefits, and the trader would choose to not invest in information. The resulting FNP plus reservation utility is $\mu = 1.5$. In contrast, as Figure 10 shows, given the same parameters the trader chooses to become informed for any fixed spread up to H-L. The implication is that constraining spreads can induce the investor to become informed, thereby enhancing price discovery, in situations where she would not choose to invest in information given competitive market making.

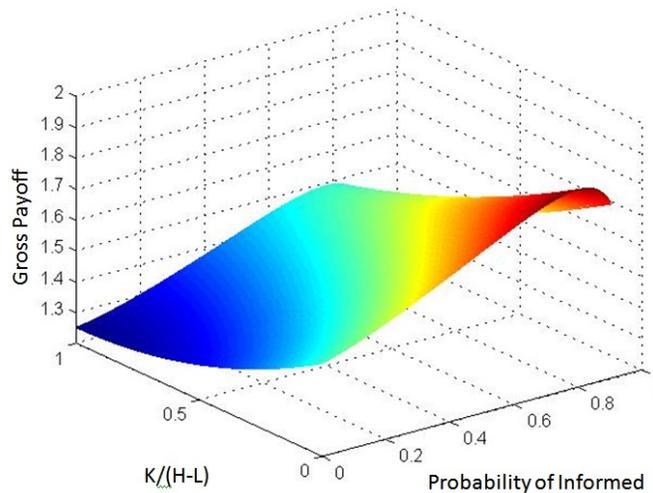


Figure 9 The investor's Gross Payoff with different level of constrained spread K and P .

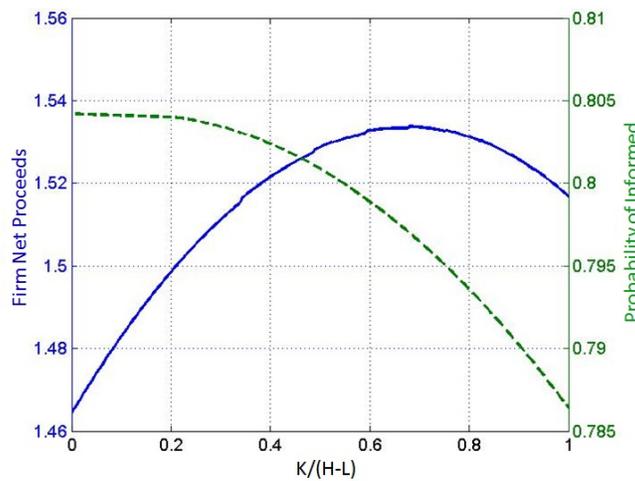


Figure 10 Maximum firm net proceeds at different level of constrained spread K with the optimal $P^*(K)$

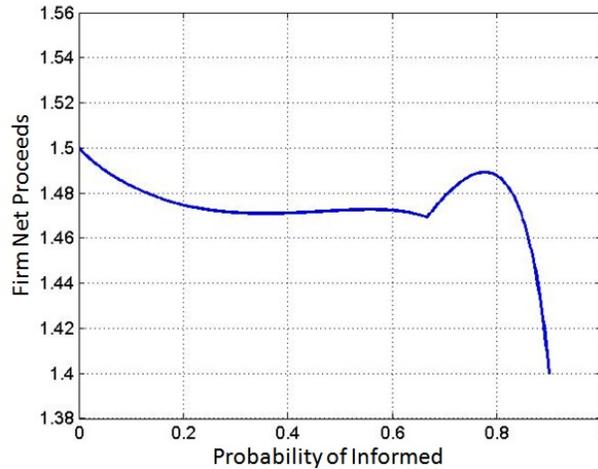


Figure 11 Firm net proceeds (equals to IPO price plus reservation utility) with competitive spread

Figures 9 to 11 are based on a specific set of parameters, including $H=2$, $L=1$, $b=0.36$, and $c=0.03$. Given these parameters, there exists an interior optimum constrained spread that maximizes firm net proceeds, and the investor chooses to incur costs in order to become informed with a constrained spread, but not with a competitive spread.

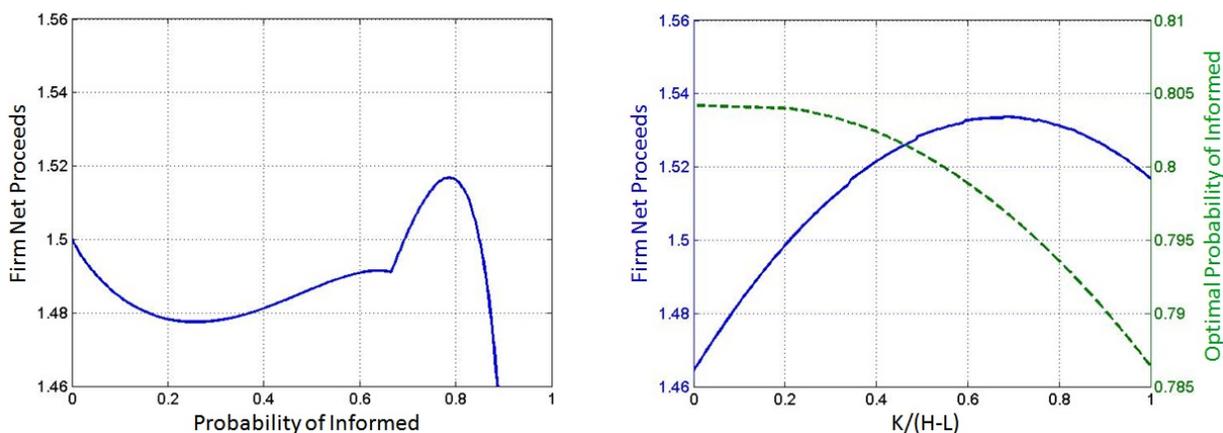
As might be anticipated, the results of this analysis are sensitive to the parameters employed, and in particular to the relative costs and benefits of information acquisition. In Figure 12 we summarize outcomes when the benefit parameter is alternately increased to $b = 0.40$ (Panel A) and decreased to $b = 0.30$ (Panel B) while the cost parameter is held constant at $c = 0.03$.

In Panel A, in the absence of a constraint on spreads, FNP plus reservation utility reaches a maximum of about 1.518 with P of about 0.8. In contrast, constraining spreads leads to net proceeds in excess of 1.52 for a wide range of constrained spreads. Given these parameters, the trader would invest in information with or without a constraint on spreads, but the firm would indeed choose to constrain the spreads to increase its net proceeds.

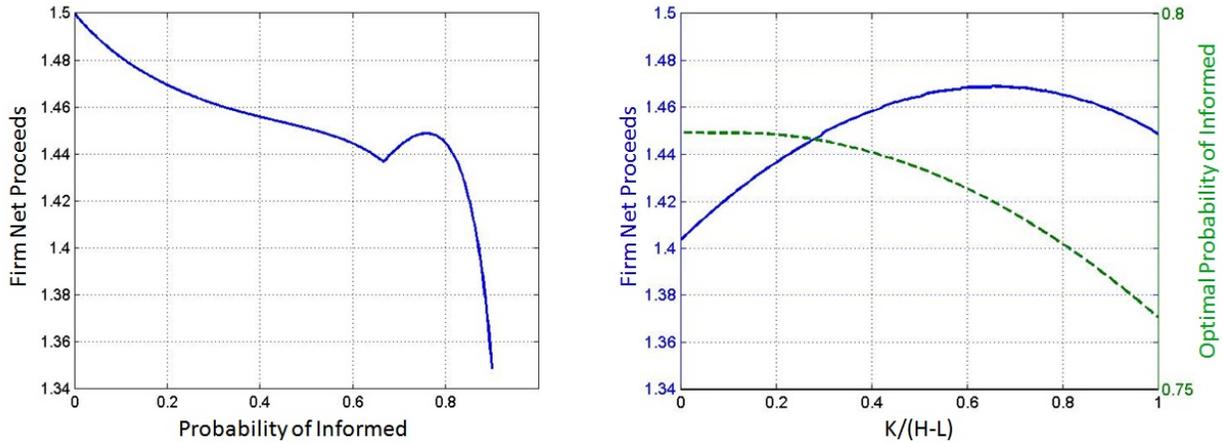
On Panel B, in the absence of a constraint on spreads, the investor would choose to not invest in private information, leading to FNP plus reservation utility of 1.5. Constraining spreads would lead the

investor to invest to increase P , but FNP plus reservation utility are less than the benchmark of 1.5 for all constrained spreads. Hence, the reduction in the benefit parameter induces the firm to not contract to narrow the spread, and as a consequence induces the trader to not invest in information.

In Section 3.4 we showed that a constraint on spread widths could always enhance value, given exogenous P . Here, we show that when becoming informed requires costly private investment, value will be enhanced and the investor will be induced to incur expenses in order to become informed, in those cases where the benefits of improved price discovery to the firm are relatively larger. To the extent that the benefits of more informative prices take the form of guidance regarding upcoming investment decisions, we anticipate that firms with greater future investment opportunities relative to assets-in-place will be more likely to contract to constrain spreads. These will tend to be young, growth firms.



(a) Firm chooses to constrain spread when benefit is relatively high. ($b=0.4$, $c=0.03$)



(b) Firm chooses not to constrain spread when benefit is relatively low. ($b=0.3$, $c=0.03$)

Figure 12 Equilibria with varying benefit parameters

4. Conclusions and Implications

We present a relatively simple model to assess the effects on firm value, gains from trade, and price discovery of a contract between a firm and a designated liquidity supplier. Our analysis shows that optimal contracted liquidity provision depends crucially on the amount of informed trading. Contracts that narrow bid-ask spreads improve economic efficiency by inducing more uninformed investors to trade. Narrower spreads also induce more informed investors to trade, which tends to improve price discovery. However, more information-based trade has a cost, which is reduced allocative efficiency. The optimal contracted bid-ask spread will reflect a balance between these considerations.

Our analysis shows that the potential benefits of contracts that require enhanced liquidity supply are related to information, rather than illiquidity, per se. The insight that seems to arise most often in contemporary discussion is that contracts for additional liquidity supply are needed for those firms or at those times when liquidity is scarce. Our analysis does not support this view, to the extent that illiquidity is attributable to high real costs, e.g. the high inventory costs associated with large volatility and/or infrequent trading. Value will not be enhanced by contracts that reduce spreads below the real social

costs of market making. Instead, our analysis implies that efficiency and firm value can be enhanced for those firms or at those times when liquidity is sparse due to real or perceived information asymmetries. This result reflects that the asymmetric information costs of providing liquidity are zero sum transfers, not net social costs. If, as Easley, Lopez de Prado, and O'Hara (2011) assert, liquidity is sometimes withdrawn from the market when perceived information asymmetries increase, our analysis implies that market quality and firm values will be enhanced by contracts that require liquidity providers to remain in the market at those times.

Our analysis is also relevant in assessing which firms would benefit from adopting a DMM agreement. Liquidity enhancement will be more valuable, *ceteris paribus*, to those firms that would benefit more from improved price discovery to guide real investment decisions. Further, liquidity enhancement will be more valuable to firms characterized by high uncertainty regarding asset value, relative to expected asset value. These will also be firms that would tend to have wider spreads in competitive equilibrium. In combination, our model implies that contracts that constrain spread widths are value enhancing at firms that (1) are characterized by non-trivial information asymmetries, (2) are subject to relatively more uncertainty regarding fundamental asset values, and (3) are likely to make substantial capital investment decisions in the reasonably near future. These are likely to be smaller, younger, and growth-oriented firms, as opposed to larger firms with a high proportion of assets-in-place.

The model presented here has a number of implications for researchers and policy makers. First, and most important, our analysis shows that market quality, economic efficiency, and firm value can be enhanced by contracts that require liquidity providers to provide more liquidity than they would otherwise choose. In light of the absence of barriers to entry in providing liquidity in electronic limit order markets, it seems reasonable to assume the business of liquidity provision is competitive. If so, side payments will be required to induce one or more designated market makers to take on such affirmative obligations, and our model implies that affirmative obligations to provide liquidity should not be imposed in the absence of compensation to the DMMs. Contracts of the type studied here, calling for a contract between the

listed firm and the DMM, are observed on some international markets, but are currently prohibited in U.S. markets by FINRA Rule 5250. Our model shows that the competitive bid-ask spread fails to maximize trader welfare or firm value, due to externalities associated with information asymmetries. DMM contracts of the type considered here comprise a potential market solution to a market imperfection.

Some limitations of our analysis should be noted. We focus exclusively on a requirement to maintain narrow spreads, and have not attempted to formally assess the optimal set of affirmative obligations. Further, as our model focuses for simplicity on a single trader, a single round of trading, and trades of unit size, we have not considered potential effects of market maker affirmative obligations on trade timing, trade sizes, repeat trading, or trading aggressiveness, each of which provides useful possibilities for future research.

Appendix

I. Investor Trading

If the investor purchases the Treasury Bill or the risky security at time 0 she has the option to trade the same asset at time 1. After her purchase of the asset and before her trading at $T=1$, she will experience a liquidity shock $\rho \sim U(-\frac{H-L}{2}, \frac{H-L}{2})$. Observing the bid and ask quotes (B and A) at time 1, the investor buys if her private value exceeds the ask, sells if her private value is below the bid, and does not trade if her subjective valuation lies between the bid and the ask. Her probability of buy, sell and no trade, and her expected payoff are assessed as follows.

1) If the investor purchases the T-bill at time 0, she can buy, sell or not trade the T-bill at zero spread at its known price at time 1. That is $A = B = V = \frac{H+L}{2}$. Her private value of the T-bill is $V^* = \frac{H+L}{2} + \rho \in (L, H)$.

$$Prob(V^* > A) = \frac{H-A}{H-L} = 0.5 \quad (A.I.1)$$

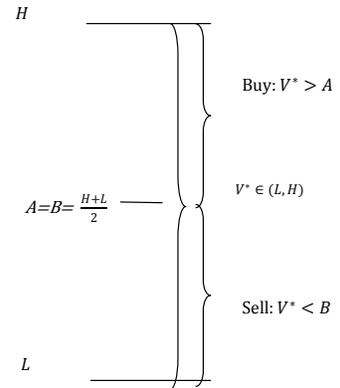
$$Prob(V^* < B) = \frac{B-L}{H-L} = 0.5 \quad (A.I.2)$$

$$Prob(A > V^* > B) = \frac{A-B}{H-L} = 0 \quad (A.I.3)$$

$$E[V^* | V^* > A] = \frac{H+A}{2} = \frac{3H+L}{4} \quad (A.I.4)$$

$$E[V^* | V^* < B] = \frac{B+L}{2} = \frac{H+3L}{4} \quad (A.I.5)$$

$$E[V^* | A > V^* > B] = \frac{H+L}{2} \quad (A.I.6)$$



2) If the investor purchases the risky security's IPO share at time 0, she will have a probability of P to become informed and she can buy, sell or not trade the risky security at (A, B). If the investor remains uninformed, her expected value of the security is $E(V) = \frac{H+L}{2}$ and her private value of the security

is $V^* = \frac{H+L}{2} + \rho \in (L, H)$.

$$Prob(V^* > A|U) = \frac{H-A}{H-L} \quad (A.I.7)$$

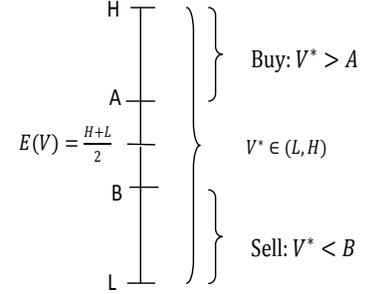
$$Prob(V^* < B|U) = \frac{B-L}{H-L} \quad (A.I.8)$$

$$Prob(A > V^* > B|U) = \frac{A-B}{H-L} \quad (A.I.9)$$

$$E[V^* | V^* > A, U] = \frac{H+A}{2} \quad (A.I.10)$$

$$E[V^* | V^* < B, U] = \frac{B+L}{2} \quad (A.I.11)$$

$$E[V^* | A > V^* > B, U] = \frac{A+B}{2} \quad (A.I.12)$$



If the investor becomes informed and learns that the true value of the security is $V=H$, then her private value of the security is $V^* = H + \rho \in (\frac{H+L}{2}, H + \frac{H-L}{2})$.

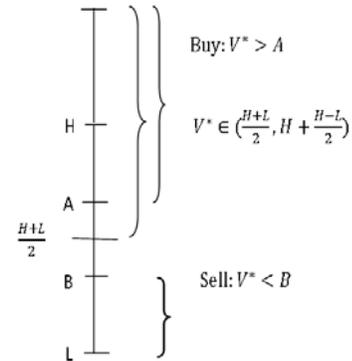
$$Prob(V^* > A|I, V = H) = \frac{H + \frac{H-L}{2} - A}{H-L} \quad (A.I.13)$$

$$Prob(V^* < B|I, V = H) = 0 \quad (A.I.14)$$

$$Prob(A > V^* > B|I, V = H) = \frac{A - \frac{H+L}{2}}{H-L} \quad (A.I.15)$$

$$E[V^* | V^* > A, I, V = H] = \frac{H + \frac{H-L}{2} + A}{2} \quad (A.I.16)$$

$$E[V^* | V^* < B, I, V = H] = 0, \quad (= \int_L^B v p(v) dv, \text{ where } p(v) = prob(v|v \in [L, B], H, I) = 0) \quad (A.I.17)$$



$$E[V^*|A > V^* > B, I, V = H] = \frac{A + \frac{H+L}{2}}{2}, (= \int_B^A v p(v) dv = \int_{\frac{H+L}{2}}^A v p(v) dp, \text{ where } p(v) = \text{prob}(v|v \in [B, A], H, I)) \quad (\text{A.I.18})$$

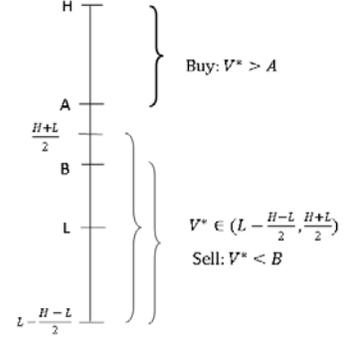
Correspondingly, if the investor becomes informed learns that the true value of the security is $V=L$, then

her private value of the security is $V^* = L + \rho \in \left(L - \frac{H-L}{2}, \frac{H+L}{2}\right)$.

$$\text{Prob}(V^* > A|I, V = L) = 0 \quad (\text{A.I.19})$$

$$\text{Prob}(V^* < B|I, V = L) = \frac{B - L + \frac{H-L}{2}}{H-L} \quad (\text{A.I.20})$$

$$\text{Prob}(A > V^* > B|I, V = L) = \frac{\frac{H+L}{2} - B}{H-L} \quad (\text{A.I.21})$$



$$E[V^*|V^* > A, I, V = L] = 0 (= \int_A^H v p(v) dv, \text{ where } p(v) = \text{prob}(v|v \in [A, H], L, I) = 0) \quad (\text{A.I.22})$$

$$E[V^*|V^* < B, I, V = L] = \frac{B + L - \frac{H-L}{2}}{2} \quad (\text{A.I.23})$$

$$E[V^*|A > V^* > B, I, V = L] = \frac{B + \frac{H+L}{2}}{2} (= \int_B^A v p(v) dv = \int_B^{\frac{H+L}{2}} v p(v) dp, \text{ where } p(v) = \text{prob}(v|v \in [B, A], H, I)) \quad (\text{A.I.24})$$

II. Bayesian Updating Based on the Observed Order (Buy, Sell, No Trade)

The market maker updates the conditional asset value on the basis of observed order using Bayes' Rule. The bid and ask quotes and the trader's decision are endogenously determined, as a lower ask implies a higher probability of buy order and a higher bid implies a high probability of sell order.

Given the probability of a buy conditional on the ask quote, the arrival of an informed trader with true value being high (or low) in eq. (A.I.13) (or in eq. (A.I.19)) and the probability of a buy conditional on the arrival of an uninformed trader in eq. (A.I.7), the probability of a buy order conditional on asset value can be stated as:

$$Prob(V^* > A|V = H) = P * Prob(V^* > A|I, V = H) + (1 - P) * Prob(V^* > A|U) \quad (A.II.1)$$

$$Prob(V^* > A|V = L) = P * Prob(V^* > A|I, V = L) + (1 - P) * Prob(V^* > A|U) \quad (A.II.2)$$

where P is the probability that the trader is informed. Upon observing a buy order, the market maker uses Bayes' Rule to update probabilities regarding the true asset value:

$$Prob(V = H|V^* > A) = \frac{Prob(V^* > A|V=H)*Prob(V=H)}{Prob(V^* > A|V=H)*Prob(V=H)+Prob(V^* > A|V=L)*Prob(V=L)} \quad (A.II.3)$$

$$Prob(V = L|V^* > A) = \frac{Prob(V^* > A|V=L)*Prob(V=L)}{Prob(V^* > A|V=H)*Prob(V=H)+Prob(V^* > A|V=L)*Prob(V=L)} \quad (A.II.4)$$

Conditional on observing a buy, the expected value of V is:

$$E(V|V^* > A) = H * Prob(V = H|V^* > A) + L * Prob(V = L|V^* > A) \quad (A.II.5)$$

When value is high (or low) and the trader is informed, the probability of a sell is given by eq. (A.I.14) (or by eq. (A.I.20)). If the trader is uninformed, the probability of a sell is in eq. (A.I.8). Therefore, the probabilities of observing a sell are given as:

$$Prob(V^* < B|V = H) = P * Prob(V^* < B|I, V = H) + (1 - P) * Prob(V^* < B|U) \quad (A.II.6)$$

$$Prob(V^* < B|V = L) = P * Prob(V^* < B|I, V = L) + (1 - P) * Prob(V^* < B|U) \quad (A.II.7)$$

where P is the probability that the trader is informed. Upon observing a sell order, the market maker uses Bayes Rule to update probabilities regarding the true asset value:

$$Prob(V = H|V^* < B) = \frac{Prob(V^* < B|V=H)*Prob(V=H)}{Prob(V^* < B|V=H)*Prob(V=H)+Prob(V^* < B|V=L)*Prob(V=L)} \quad (A.II.8)$$

$$Prob(V = L|V^* < B) = \frac{Prob(V^* < B|V=L)*Prob(V=L)}{Prob(V^* < B|V=H)*Prob(V=H)+Prob(V^* < B|V=L)*Prob(V=L)} \quad (A.II.9)$$

Conditional on a sell, the market maker updates the expected value of V as:

$$E(V|V^* < B) = H * Prob(V = H|V^* < B) + L * Prob(V = L|V^* < B). \quad (A.II.10)$$

The absence of a trade is also potentially informative regarding firm value. If the trader is informed, the probability of no trade is given by eq. (A.I.15) when V=H and eq. (A.I.21) when V=L. If the trader is uninformed, the probability of no trade is given by in eq. (A.I.9). Therefore, conditional no-trade probabilities are:

$$Prob(A > V^* > B|V = H) = P * Prob(A > V^* > B|I, V = H) + (1 - P) * Prob(A > V^* > B|U) \quad (A.II.11)$$

and

$$Prob(A > V^* > B|V = L) = P * Prob(A > V^* > B|I, V = L) + (1 - P) * Prob(A > V^* > B|U) \quad (A.II.12)$$

And probabilities regarding asset values conditional on no trade are given as:

$$Prob(V = H|A > V^* > B) = \frac{Prob(A > V^* > B|V=H)*Prob(V=H)}{Prob(A > V^* > B|V=H)*Prob(V=H) + Prob(A > V^* > B|V=L)*Prob(V=L)} \quad (A.II.13)$$

$$Prob(V = L|A > V^* > B) = \frac{Prob(A > V^* > B|V=L)*Prob(V=L)}{Prob(A > V^* > B|V=H)*Prob(V=H) + Prob(A > V^* > B|V=L)*Prob(V=L)} \quad (A.II.14)$$

Conditional on the no-trade outcome, the market maker will update the expected value of V as the following:

$$E(V|A > V^* > B) = H * Prob(V = H|A > V^* > B) + L * Prob(V = L|A > V^* > B) \quad (A.II.15)$$

With a given spread, the unconditional probabilities of a buy, sell, and no trade are:

$$Prob(V^* > A) = PH * Prob(V^* > A|V = H) + PL * Prob(V^* > A|V = L) \quad (A.II.16)$$

$$Prob(V^* < B) = PH * Prob(V^* < B|V = H) + PL * Prob(V^* < B|V = L) \quad (A.II.17)$$

$$Prob(A > V^* > B) = PH * Prob(A > V^* > B|V = H) + PL * Prob(A > V^* > B|V = L) \quad (A.II.18)$$

where $Prob(V^* > A|V = H)$, $Prob(V^* < B|V = H)$, $Prob(A > V^* > B|V = H)$, $Prob(V^* > A|V = L)$, $Prob(V^* < B|V = L)$, and $Prob(A > V^* > B|V = L)$ are given in eq. (A.II.1), (A.II.6), (A.II.11), (A.II.2), (A.II. 7). and (A.II.12).

III. Proof of Lemmas and Propositions

Proof of Lemma 1: We determine the competitive bid and ask quotes based on a zero-expected profit condition, similar to Glosten and Milgrom (1985). The market maker sets the zero profit ask price (A) to the conditional expected value based on a buy and the zero profit bid price (B) to the conditional expected value based on a sell as the followings:

$$A = E(V|V^* > A) \tag{A.III.1}$$

and

$$B = E(V|V^* < B), \tag{A.III.2}$$

where $E(V|V^* > A)$ and $E(V|V^* < B)$ are given in eq. (A.II.5) and eq. (A.II.10). Given the distributional assumption of the liquidity shock $\rho \sim U\left(-\frac{H-L}{2}, \frac{H-L}{2}\right)$, $\text{Prob}(V = L) = \text{Prob}(V = H) = 0.5$, and the exogenous P , the competitive bid and ask price can be obtained by solving $A = E(V|V^* > A)$ and $B = E(V|V^* < B)$ respectively,

For P less than $2/3$:

$$\text{The competitive ask price } A = \frac{0.5*(P*H+2H+2L-3P*L)}{2-P}$$

$$\text{The competitive ask price } B = \frac{0.5*(P*L+2H+2L-3P*H)}{2-P}$$

For P greater than or equal to $2/3$:

$$\text{The competitive ask price } A = H$$

$$\text{The competitive ask price } B = L$$

Following section 3.3 table 2, from the viewpoint of the investor, the unconditional expectation of the investor's net gain from participating in the IPO(column iii), given time 1 trading, can be written as:

$$\begin{aligned} \pi = & \text{Prob}(V^* > A)[2E(V^*|V^* > A) - (A + Q)] + \\ & \text{Prob}(B < V^* < A)[E(V^*|(B < V^* < A)) - Q] + \text{Prob}(V^* < B)[B - Q]. \end{aligned}$$

As noted, the investor becomes informed regarding the final value of the asset, prior to time 1 trading, with probability P . Differentiating between the case where the investor is informed or not, the unconditional expectation as of time zero of the net gain can be expressed as:

$$\begin{aligned}
\pi = & (1 - P) \times \{ \text{Prob}(V^* > A|U)[2 \times E(V^*|V^* > A, U) - (A + Q)] \\
& + \text{Prob}(B < V^* < A|U)[E(V^*|B < V^* < A, U) - Q] \\
& + \text{Prob}(V^* < B|U)[B - Q] \} \\
& + 0.5 \times P \times \{ \text{Prob}(V^* > A|I, V = H)[2 \times E(V^*|V^* > A, I, V = H) - (A + Q)] \\
& + \text{Prob}(B < V^* < A|I, V = H)[E(V^*|B < V^* < A, I, V = H) - Q] \\
& + \text{Prob}(V^* < B|I, V = H)[B - Q] \} \\
& + 0.5 \times P \times \{ \text{Prob}(V^* > A|I, V = L)[2 \times E(V^*|V^* > A, I, V = L) - (A + Q)] \\
& + \text{Prob}(B < V^* < A|I, V = L)[E(V^*|B < V^* < A, I, V = L) - Q] \\
& + \text{Prob}(V^* < B|I, V = L)[B - Q] \} .
\end{aligned}$$

where the conditional probabilities $\text{Prob}(V^* > A|U)$, $\text{Prob}(V^* < B|U)$, and $\text{Prob}(B < V^* < A|U)$ are given in eq. (A.I.7), (A.I.8) and (A.I.9). $\text{Prob}(V^* > A|I, V = H)$, $\text{Prob}(B < V^* < A|I, V = H)$, and $\text{Prob}(V^* < B|I, V = H)$ are given in eq. (A.I.13), (A.I.14) and (A.I.15). $\text{Prob}(V^* > A|I, V = L)$, $\text{Prob}(B < V^* < A|I, V = L)$, and $\text{Prob}(V^* < B|I, V = L)$ is given in eq. (A.I.19), (A.I.20) and (A.I.21) respectively. The conditional expected private values $E(V^*|V^* > A, U)$ and $E(V^*|B < V^* < A, U)$ are given in (A.I.10) and (A.I.12). $E(V^*|V^* > A, I, V = H)$, $E(V^*|B < V^* < A, I, V = H)$, $E(V^*|V^* > A, I, V = L)$, and $E(V^*|B < V^* < A, I, V = L)$ are given in eq. (A.I.16), (A.I.18), (A.I.22) and (A.I.24) respectively.

The firm's equilibrium initial offer price Q can be derived from the participation constraint of the investor. To induce the investor to participate, the firm chooses Q so that the expected payoff π equals π_{tbill} .

When $P < 2/3$, given the zero profit bid and ask spreads, the firm will set the offer price of the risky security as $Q^l = \mu - \frac{P(H-L)}{4}$. When $P > 2/3$, given the zero profit bid and ask spreads, the firm will set the offer price of the risky security as $Q^h = \mu - \frac{(2-P)(H-L)}{8}$. ■

Proof of Proposition 1: When $P \leq \frac{2}{3}$, $Q_K - Q = \frac{(2-P)}{8(H-L)}K^2 - \frac{K}{2} + \frac{P(H-L)}{2}$. Then,

$$\frac{\partial Q_K - Q}{\partial K} = \frac{K(2-P)}{4(H-L)} - \frac{1}{2} < \frac{\frac{2P(H-L)}{2-P}(2-P)}{4(H-L)} - \frac{1}{2} = \frac{1}{2}(P - 1) < 0,$$

where the first inequality makes use of the fact that $K < \frac{2P(H-L)}{2-P}$, i.e. that the constrained spread is less than the unconstrained spread.

When $P \geq \frac{2}{3}$, $Q_K - Q = \frac{(2-P)}{8(H-L)}K^2 - \frac{K}{2} + \frac{(2+P)(H-L)}{8}$. Then,

$$\frac{\partial Q_K - Q}{\partial K} = \frac{K(2-P)}{4(H-L)} - \frac{1}{2} < \frac{(H-L)(2-P)}{4(H-L)} - \frac{1}{2} \leq \frac{2-\frac{2}{3}}{4} - \frac{1}{2} = -\frac{1}{6} < 0,$$

where the first inequality makes use of the fact that $K < H - L$, i.e. that the constrained spread is less than the unconstrained spread. ■

Proof of Lemma 3: Given any quote (A, B) , the liquidity provider's unconditional expectation of market making loss can be stated as:

$$M = \text{Prob}(V^* > A)[E(V|V^* > A) - A] + \text{Prob}(V^* < B)[B - E(V|V^* < B)]$$

Where $Prob(V^* > A)$ and $Prob(V^* < B)$ are given in eq. (A.II.16) and (A.II.17) respectively. $E(V|V^* > A)$ and $E(V|V^* < B)$ are given in eq. (A.II.5) and (A.II.10) respectively.

By definition, the market maker's loss is zero with the competitive spread, where $A = E(V|V^* > A)$ and $B = E(V|V^* < B)$. When the spread is constrained to K , such that $A = \frac{H+L}{2} + \frac{K}{2}$ and $B = \frac{H+L}{2} - \frac{K}{2}$, the liquidity provider's expected market making loss, M , which also equals the requisite payment from the firm to the market makers, is given as:

$$M = \frac{(2-P)}{4(H-L)} K^2 - \frac{(2+P)}{4} K + \frac{P(H-L)}{2}.$$

The derivative with respect to the spread constraint K is $\frac{\partial M}{\partial K} = \frac{(H-L)(P+2) - 2K(2-P)}{-4(H-L)}$.

When $P \leq \frac{2}{3}$, $\frac{\partial M}{\partial K} < \frac{(H-L)(P+2) - 2 \frac{2P(H-L)}{2-P} (2-P)}{-4(H-L)} = \frac{(2-3P)(H-L)}{-4(H-L)} \leq 0$, where the first inequality follows from the fact that $K < \frac{2P(H-L)}{2-P}$, i.e. that the constrained spread is less than the competitive spread.

When $P \geq \frac{2}{3}$, $\frac{\partial M}{\partial K} < \frac{(H-L)(P+2) - 2(H-L)(2-P)}{-4(H-L)} = \frac{3P-2}{-4} \leq 0$, where the first inequality follows from the fact that $K \leq H - L$, i.e. that the constrained spread is less than the competitive spread.

Proof of Proposition 2: The derivative of the firm's net proceeds with respect to K is: $\frac{\partial(Q_K - P)}{\partial K} = \frac{P(H-L) - K(2-P)}{4(H-L)}$. Setting this first order condition to zero and solving for K , we obtain the optimal spread

$K^* = \frac{P(H-L)}{2-P}$. Substituting the expression for K^* into the expressions for the change in firm net proceeds,

we have:

When $P < 2/3$, $(Q_K - Q^l) - M = \frac{P^2 * (H-L)}{8 * (2-P)} \geq 0$, and

when $P > 2/3$, $(Q_K - Q^h) - M = \frac{(1-P)^2 * (H-L)}{2 * (2-P)} \geq 0$. ■

Proof of Proposition 3: When $P \leq \frac{2}{3}$, $(Q_K - Q) - M = -\frac{(2-P)}{8(H-L)} K^2 + \frac{P}{4} K = K \left(\frac{P}{4} - \frac{(2-P)}{8(H-L)} K \right) >$

$K \left(\frac{P}{4} - \frac{(2-P)}{8(H-L)} \frac{P(H-L)}{2-P} \right) = 0$; where the inequality follows from the fact that $K < \frac{2P(H-L)}{2-P}$, i.e. that the

constrained spread is less than the competitive spread.

When $P \geq 2/3$, $(Q_K - Q) - M = -\frac{(2-P)}{8(H-L)} K^2 + \frac{P}{4} K + \frac{(H-L)(2-3P)}{8}$.

That is $(Q_K - Q) - M > 0$ when $\left| K - \frac{P(H-L)}{2-P} \right| < \frac{2(1-P)(H-L)}{2-P}$.

Therefore, if $\frac{(3P-2)(H-L)}{2-P} < K < \frac{2P(H-L)}{2-P}$, then $(Q_K - Q) - M > 0$.

If $0 < K \leq \frac{(3P-2)(H-L)}{2-P}$, then $(Q_K - Q) - M \leq 0$. ■

IV. Combined Gains from Trade

Referring to column (x) of Table 2 in section 3.3, the unconditional expectation of the combined gains from $t=1$ trading can be expressed as:

Combined Gain from Trading (CGT)

$$= Prob(V^* > A) * E(\rho | V^* > A) + Prob(V^* < B) * E(-\rho | V^* < B)$$

As noted, the investor becomes informed regarding the final value of the asset, prior to time 1 trading, with probability P . Differentiating between the case where the investor is informed or not, the unconditional expectation of the combined gains can be expressed as:

$$CGT = (1 - P) \times \{ Prob(V^* > A | U) [E(V^* | V^* > A, U) - A - E(V)]$$

$$\begin{aligned}
& + Prob(V^* < B|U)[B - E(V^*|V^* < B, U) - E(V)] \} \\
& + 0.5 \times P \times \{ Prob(V^* > A|I, V = H)[E(V^*|V^* > A, I, V = H) - A - H] \\
& \quad + Prob(V^* < B|I, V = H)[B - E(V^*|V^* < B, I, V = H) - H] \} \\
& + 0.5 \times P \times \{ Prob(V^* > A|I, V = L)[E(V^*|V^* > A, I, V = L) - A - L] \\
& + Prob(V^* < B|I, V = L)[B - E(V^*|V^* < B, I, V = L) - L] \} .
\end{aligned}$$

Where the conditional probabilities $Prob(V^* > A|U)$ and $Prob(V^* < B|U)$ are given in eq. (A.I.7) and (A.I.8). $Prob(V^* > A|I, V = H)$ and $Prob(V^* < B|I, V = H)$ are given in eq. (A.I.13) and (A.I.14). $Prob(V^* > A|I, V = L)$ and $Prob(V^* < B|I, V = L)$ are given in eq. (A.I.19) and (A.I.20). The conditional expected private values $E(V^*|V^* > A, U)$, $E(V^*|V^* > A, I, V = H)$, $E(V^*|V^* > A, I, V = L)$ are given in eq. (A.I.10). (A.I.16) and (A.I.22). $E(V^*|V^* < B, U)$, $E(V^*|V^* < B, I, V = H)$, $E(V^*|V^* < B, I, V = L)$ are given in eq. (A.I.11). (A.I.17) and (A.I.23).

Given the distributional assumption of the liquidity shock $\rho \sim U\left(-\frac{H-L}{2}, \frac{H-L}{2}\right)$, the combined gain from trade with the constrained spread, K , can be further simplified to: $CGT = \frac{P-2}{8(H-L)}K^2 + \frac{P}{4}K + \frac{(H-L)-P(H-L)}{4}$.

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